APRIL 2005
VOL 6, NO 8

TAX • REGULATION • FINANCE

DERIVATIVES

FINANCIAL PRODUCTS REPORT

DERIVATIVES AND THE DEMAND FOR FINANCIAL MATH—IT *IS* ROCKET SCIENCE

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THE TRENDS POINT TO ONE UNDENIABLE TRUTH: IN THE FUTURE, FINANCIAL MATHEMATICS GRADUATES WILL BE IN DEMAND.

An MIT physics professor recently noted that the most remarkable change he has witnessed during the past four decades is the number of physics and math majors going into the field of finance. "Even ten years ago you just didn't see this," he said, adding that, "never in my wildest dreams would I have predicted this."

While an exact explanation for the change is difficult to provide, two key trends may be relevant. The first is the explosion in the growth of the derivative markets during the 1990s, and the other is the availability of a new kind of education especially suited to undergraduates with excellent training in mathematical and computational skills: the Masters Degree in Financial Mathematics.

In the fall of 1996, 25 math, physics, and computer science graduates entered the inaugural class of the University of Chicago's Master of Science in Mathematical Finance Program. The one-year program, whose content and curriculum were scrupulously researched in collaboration with several industry experts, was intended to train students in the methods of mathematics as they are applied to finance. The program as it was in 1996 was approximately one-half mathematics courses, such as stochastic calculus, partial differential equations, and optimization theory, and one-half finance, including courses in derivative markets and instruments as well as heavily theoretical courses in the mathematics of derivatives pricing. At the time, almost no programs of this kind existed, and it was questionable whether Chicago's new program made it one too many.

It was not that high-powered mathematics was not already an important part of the world of finance. Rather, high-powered financial mathematics was the province of "rocket scientists," that is, math, physics, and engineering Ph.Ds with research training in mathematical and physical sciences. The issue was whether the industry had any use for Masters students and whether they could compete with the myriad newly minted Ph.Ds who were regularly finding their way to Wall Street. When the University of Chicago started to research its Mathematics of Finance Program in 1994, much of the feedback that the school received was along the lines of "great program, I don't know if your students are going to get jobs."

Today, eight years later, the International Association of Financial Engineers lists over 70 Masters in Financial Mathematics programs on its website, and the number of such programs has been growing at an astounding rate.[1] Many of the world's top universities now offer financial mathematics programs. Besides the University of Chicago, these include Princeton, Stanford, MIT, Columbia, Cambridge, and Oxford. Each year the International Association of Financial Engineers holds its National Financial Mathematics Career Fair in New York City and financial giants such as Goldman Sachs, Citigroup, and Merrill Lynch regularly attend this meeting to recruit graduates from 20 different math finance programs.[2]

At its core, the programs' popularity and growing importance can be explained quite simply by the incredible and surprising usefulness of mathematics in finance, especially in financial derivatives. While this area became an important part of the financial landscape in the 1970s, it truly came into its own during the 1990s.

In March 1999, Alan Greenspan, Chairman of the Federal Reserve Board, speaking before the Futures Industry Association, opened with this statement: "By far the most significant event in finance during the past decade has been the extraordinary development and expansion of financial derivatives." He credits derivatives for their unique ability to break up risk into component parts and trade them with counterparties more willing and able to take on those risks. As he noted:

> This unbundling improves the ability of the market to engender a set of product and asset prices far more calibrated to the value preferences of consumers than was possible before derivative markets were developed. The product and asset price signals enable entrepreneurs to finely allocate real capital facilities to produce those goods and services most valued by consumers, a process that has undoubtedly improved national productivity growth and standards of living.

What Greenspan did not say is that the tremendous growth, which has continued at the same rate into this decade, has created tremendous demand for highly trained individuals capable of understanding and improving the underlying theoretical models for pricing and hedging the derivative instruments. This is because, when it comes to the pricing, hedging, and risk management of financial derivatives, sophisticated mathematics is not optional; it is required reading. The creation of new products, their design, and implementation—from the creation of risk systems to hedging schemes to research pieces that educate clients—requires exactly the kind of knowledge and background that financial mathematics graduates have. And this knowledge is not easy to come by, even for math and physics Ph.Ds. Hence the need for highly specialized training in financial mathematics.

With this, one might think that financial mathematics programs are merely professional schools for would-be, want-to-be derivatives traders and sales people. In fact, the ambitions of financial mathematics programs extend beyond derivatives to the broader world of finance, where another exciting development in finance during the past decade comes into play. Enter quantitative investment strategies.

A major investment bank recently posted the following job description under the heading "Quantitative Trading and Research":

> As products in the financial markets become more sophisticated and proprietary trading risks increase, the successful integration of quantitative methods with trading takes on a greater significance. Quantitative traders are primarily focused on providing capital and managing risk on customer trading desks such as Mortgages, Fixed Income, Equity, Foreign Exchange (FX), and Credit Derivatives. The rise in prominence of proprietary trading also has led to the development of increasingly sophisticated trading techniques designed to identify market inefficiencies in Statistical Equity Arbitrage, Fixed Income Arbitrage, and Municipal Arbitrage.

Trading strategies based on quantitative techniques are not new to Wall Street. Indeed, it is widely known that David Shaw, founder of the hedge fund giant D.E. Shaw, worked in a highly quantitative trading group in the mid-to-late 1980s before leaving to start his own fund. What is new, however, is the explo-

[1] See www.iafe.org/?id=academicprograms.

[2] For more information, see www.iafe.org/events.php?event_id=34972567.

sive growth of hedge funds and trading groups within investment banks that specialize in deploying capital using sophisticated statistical analysis.

What has made this possible is the rapid decline in both the cost of computing power and storage. Both components are necessary to manipulate the vast amounts of data available today. A typical quantitatively driven hedge fund can easily possess multiple "terabytes" of data on global markets. The ability to store and rapidly retrieve such large amounts of data requires computers with large amounts of on-board RAM, fast processors, and lots and lots of disk space.

What does one do with all this computing power and data? Two words: *analyze it*.

FINANCIAL MATHEMATICS

Financial mathematics spans several subject areas crossing various disciplines. To be truly effective, students must learn everything from the mathematics related to partial differential equations, stochastic processes, and optimization to econometrics, data analysis, and numerical analysis at the same time they are mastering the intricacies of the uses of financial derivatives across several different markets. The mathematics in financial math is essential to understanding the pricing models that assign fair values to financial derivatives.

Equally important to the theory is the ability to implement the models into practical applications for analyzing the profitability and risk of particular derivative transactions and portfolios of derivatives. These practical applications make the models accessible to traders and sales people who need to communicate the outputs of such models. In this regard, most financial mathematics programs emphasize computer implementation of financial models in languages like C++ and Java.

Alongside the mathematical theory and computer implementation of financial mathematics, programs also require courses in financial markets, financial instruments, and their uses in the "real world" to train students in the practical workings of global markets and the instruments that trade on them. They bridge theory with practice by explaining the mechanics of trading and the uses for the products, expanding, if you will, on Greenspan's remarks above. The best financial mathematics programs are able to leverage their reputations to attract world-class practitioners to participate in the teaching of courses to give students a real-world look at the use of such instruments.

WHAT IS NEW IS THE EXPLOSIVE GROWTH OF HEDGE FUNDS AND TRADING GROUPS WITHIN INVESTMENT BANKS THAT SPECIALIZE IN DEPLOYING CAPITAL USING SOPHISTICATED STATISTICAL ANALYSIS.

FINANCIAL DERIVATIVES

Derivatives are financial instruments whose value and payouts depend on the movement of other financial variables. This simple statement belies both the usefulness and complexity of derivatives. But considering that until the mathematical sophistication of the instruments was first truly understood—studied in the seminal work of Fischer Black, Myron S. Scholes, and Robert C. Merton in 1972 for which Scholes and Merton later (1997) won the Nobel prize—derivatives were literally nothing more than a backwater business of little significance to the financial world.

This changed in the early 1970s. First, Black, Scholes, and Merton published their work on option pricing theory[3] and demonstrated that the value of a call option on a company's stock is determined by the solution to a certain partial differential equation. Second, the Chicago Board Options Exchange opened its doors. To understand the importance of Black, Scholes, and Merton, all one has to do know is that, within six months of the options exchange's opening, Texas Instruments was offering a hand-held calculator with the Black-Scholes formula programmed into it.

It is indisputable that, from the early 1970s to today, derivative instruments have been one of the most successful new products in any industry since the invention of the automobile. From essentially zero sales in 1970, derivatives have grown to be a multi-trillion dollar industry and are still sustaining double-digit growth 30 years later. What drives this growth is their fundamental importance to end-users—hedge funds, commercial banks, corporate treasuries, and central banks—in hedging financial risks. The ability for any of these to hedge such risks expands their fundamental ability to operate their businesses. Markets for these instruments allow not only for better management of risks—that is, the hedging of risks that were previously unhedgeable—but for the measurement of risks that were previously immeasurable. The dual ability to measure and manage financial risk is of unparalleled importance in financial markets.

Derivatives come in a variety of sizes and shapes, but one way to categorize them is along asset class lines. There are interest rate derivatives (e.g., an interest rate swap may pay a customer according to the current level of short-term interest rates and be used to hedge risks in a shifting interest rate environment), and there are equity derivatives (e.g., a put option might pay a customer according to the value of the S&P 500 index and be used to hedge market risk exposure). There are also foreign exchange derivatives (e.g., foreign exchange options), which pay off according to underlying moves in FX markets.

Finally, newer but perhaps most important, are credit derivatives. A credit default swaps pays a customer in the event of a certain credit defaulting. All of these instruments play an increasingly important role in the financial marketplace as banks offer an increasingly complicated variety of such instruments to a widening customer base. Summarizing all of this, Alan Greenspan recently remarked:

> No discussion of better risk management would be complete without mentioning derivatives and the technologies that spawned them and so many other changes in banking and finance. Derivatives have permitted financial risks to be unbundled in ways that have facilitated both their measurement and their management. Because risks *can* be unbundled, individual financial instruments can now be analyzed in terms of their common underlying risk factors, and risks can be managed on a portfolio basis. Concentrations of risk are more readily identified, and when such concentrations exceed the risk appetites of intermediaries, derivatives and other credit and interest rate risk instruments can be employed to transfer the underlying risks to other entities. As a result, not only have individual financial institutions become less vulnerable to shocks from underlying risk factors, but also the financial system as a whole has become more resilient.[4]

The growth of derivatives was tremendous during the 1990s but has continued unabated in the 21st century. The BIS (Bank for International Settlements) tracks OTC derivative markets in a once-every-three-years survey of derivatives dealers globally. From 1995 to 2004, the daily turnover in interest rate derivatives grew steadily from $151 billion to $1.025 trillion, representing an annual rate of increase of over 21%.[5] This is startling, especially since interest rate derivatives were already considered a mature industry. Similar huge growth occurred in foreign exchange, equity, and credit derivatives as well.

Derivatives markets are composed of a variety of players but central to it are the derivatives dealers—that is, investment banks whose traders stand ready to buy or sell in a variety of instruments. These banks, such as Goldman Sachs, Morgan Stanley, UBS, and Credit Suisse First Boston, are usually organized along asset class lines with the derivatives traders situated physically close to the traders in the underlying instruments of the asset classes. So, for example, an equity derivatives trader sits on the equity trading floor near the block traders, program traders, and Nasdaq market makers. The derivatives desk consists of traders, sales people, and a quantitative team. Traders make the markets—they say at what price they will buy and where they will sell a certain instrument on a certain day, at a certain time. Sales people keep in close contact with their customers and take requests for pricing information on derivatives as well as pitching trade ideas. When a customer wants to do a trade, he communicates the request to the sales person who then asks the trader to provide a two-way market—a price at which he will buy and a price at which he will sell. From here, after a bit of routine discussion, a trade is executed.

WITHIN SIX MONTHS OF THE OPENING OF THE CHICAGO BOARD OPTIONS EXCHANGE, TEXAS INSTRUMENTS WAS OFFERING A HAND-HELD CALCULATOR WITH THE BLACK-SCHOLES FORMULA PROGRAMMED INTO IT.

All of this involves a tremendous amount of mathematical, financial, and technological sophistication. Many of the instruments that a derivatives desk offers are OTC derivatives, meaning that a trade is a private contractual arrangement between a customer and a trading desk. It involves an obligation by both parties to pay certain cashflows in the event of certain price movements. For example, in an equity option, the bank may agree to pay the counterparty if the price of the underlying instrument (e.g., the S&P 500) falls below a certain level. The bank, naturally, does not want to

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3 Black and Scholes, "The Pricing of Options and Corporate Liabilities," 81 Journal of Political Economy 637 (1973). Merton, "Theory of Rational Option Pricing," 4 Journal of Economics and Management Science 141 (1973).

4 "Banking," remarks by Chairman Alan Greenspan at the American Bankers Association Annual Convention, New York, New York, October 5, 2004 (http://www.federalreserve.gov/boarddocs/speeches/2004/20041005/default.htm).

5 See Bank for International Settlements, "Triennial Central Bank Survey of Foreign Exchange and Derivative Markets Activity in April of 2004," at http://www.bis.org/publ/rpfx04.pdf.

play the role of casino but instead wants to individually hedge each trade to ensure that each and every trade turns a profit. This is where financial mathematics plays its biggest role.

It was precisely the work of Black, Scholes, and Merton that demonstrated that not only does a precise mathematical relationship exist between the value of a derivative instrument and the underlying economic variables that influence its price, but also that there is a precise way of hedging a derivative transaction. Thus, if the formula states that a certain option is worth $1, a trader can sell the instrument for $1.10, hedge it, and book a $0.10 profit. The mathematics behind this are based on a financial notion called "arbitrage-free pricing," which essentially says that the law of one price must always hold. It recognizes that two financially equivalent bundles of goods, no matter how differently they are bundled, must have the same price. From this simple observation and some sophisticated mathematics, a hedging scheme for stock options came about, and every new derivatives pricing scheme since has been based on this idea.

To provide a two-way market on a derivative instrument, a trader must compute the fair price of the derivative and understand the hedging scheme that allows the realization of that price. The pricing formulas must therefore be embedded in technology that exposes to the trader live market prices and pricing models for a variety of derivative instruments. The trader must also be able to confer with a quantitative specialist to help interpret the validity of the prices and hedges in various market situations. After all, the aim of the trader is to offer the financial instrument to the customer and turn a profit as the market evolves in unknown ways.

Months if not years before, the derivatives group may decide that a demand exists for a new derivative product due to client inquiries, news of competitors offering such a product, or the brainchild of the sales team brainstorming with the quantitative team. Whatever the origin, the question then arises whether this product can be offered and can be profitable. To answer this requires modeling.

FROM 1995 TO 2004, THE DAILY TURNOVER IN INTEREST RATE DERIVATIVES GREW STEADILY FROM $151 BILLION TO $1.025 TRILLION, REPRESENTING AN ANNUAL RATE OF INCREASE OF OVER 21%.

The quantitative team must devise mathematical models that relate the underlying instrument variables to the value of the derivative contract. These models must be thoroughly tested in a variety of scenarios and under a variety of circumstances. The whole team must be educated on how the models work. Then the quantitative team must build these pricing models into the analytical tools used by the traders and sales people. After that, the team must put together marketing pieces describing the instruments and their models to give to their customers. The underlying message is always "we have the know-how and brainpower to offer you these new financial instruments."

A derivative group's profits are ultimately determined by how successfully they market their offerings to their client base, how successfully they can hedge the risks of the products they offer, and finally how many useful new, high-margin products they can deliver to their clients. The number of banks that offer derivatives is large, and they all need people trained in the mathematics of derivatives.

QUANTITATIVE TRADING

If mathematics is an essential tool in one area of finance, it is reasonable that it should be in other areas. Indeed, while financial mathematics has become a completely integrated part of derivatives trading, it is a significant, if not essential, part of many other financial trading operations. In particular, an increasing percentage of asset management companies are devoting efforts to quantitative trading strategies, and an increasing number of brokerage houses are using mathematics in a fundamental way to improve their trading businesses.

Quantitative asset management refers to the practice of using statistical and mathematical research to build profitable trading strategies. There is no single type of quantitative trading strategy; they span different markets and different asset classes. In equity trading, these strategies are usually characterized by large trading books, with hundreds if not thousands of stocks being simultaneously held long and short. In non-equity related trading, such as currencies, fixed income, and commodities, quantitative portfolio managers use sophisticated valuation models to identify overvalued and undervalued assets. In both instances, the name of the game is conducting research seeking an edge in trading each asset and then exploiting that advantage by means of very precise trading methods.

What is needed is a blend of mathematical and statistical know-how and, in many instances, excellent computing skills to automate the trading. Financial theory must be blended with the ability to draw conclusions from messy and sometimes conflicting data. To be really good, nerves of steel are needed to commit capital to one's conclusions and then live or die by the results.

Sometimes, the outcomes have been quite impressive. For example, Renaissance Technologies' Medallion International Fund has returned more than 35% on average per year since its inception in 1988.[6] This is, according to some, the single greatest hedge fund achievement in the history of funds.[7] Renaissance Technologies trading strategies are completely quantitative.

D.E. Shaw was founded in 1988 by David E. Shaw and has a long history of success using quantitative methods to make money. In fact, D. E. Shaw's website devotes an entire section to quantitative trading, stating:

> Underpinning many aspects of the firm's investment and financial service activities is a body of mathematical and computational techniques developed in the course of several hundred person-years of analytical and experimental research.... Such techniques are employed not only to identify investment opportunities, but also for the purposes of portfolio optimization, risk management and the minimization of transaction costs.[8]

The University of Chicago believes that quantitative trading is one of the most important future directions for financial mathematics and is devoting considerable resources to sharpening its curriculum's focus on such areas. To that end, its Masters in Financial Mathematics Program just appointed recently retired Renaissance Managing Director, Robert Frey, to head its advisory board and D.E. Shaw's Head of Proprietary Trading, Eric Wepsic, as a board member.

Quantitative asset management is only one interpretation of quantitative trading. Another huge area where mathematics and trading are colliding is in Wall Street's biggest new trading business, algorithmic trading. "Algorithmic trading" refers to automated trading "machines" that take customer orders for large baskets of stocks or large stock trades and use mathematical models to optimally trade those orders using less human interaction (read: less expense). In a recent article in *Institutional Investor*, Justin Schack outlines the fiercely competitive institutional trading business' search to improve margins and shore up the traditionally profitable business:

RENAISSANCE TECHNOLOGIES' MEDALLION INTERNATIONAL FUND HAS RETURNED MORE THAN 35% ON AVERAGE PER YEAR SINCE ITS INCEPTION IN 1988.

> One solution: algorithmic trading platforms. Firms like Goldman, through its Spear, Leeds & Kellog division, and CSFB have long been leaders in this realm. Now rivals like Citi and Merrill are devoting more resources to this low-touch business.... One newer algorithm, known as "arrival price" or "implementation shortfall" sets a time limit for execution and seeks to get as much of an order possible executed at the price where the stock was trading when the order was executed. Only the most advanced platforms...offer arrival price algorithms.[9]

Behind these algorithms are naturally sophisticated mathematics that require the research, analytical, and implementation skills of, among others, financial mathematics graduates. In fact, the research ideas behind algorithmic trading were developed by Neil Chriss, who in addition to managing money at SAC Capital is also Executive Director of the Financial Mathematics Program at University of Chicago, and Robert Almgren, who is Director of the Financial Mathematics Program at University of Toronto.[10] In fact, *Institutional Investor* goes so far as to say that their paper "helped lay the groundwork for arrival-price algorithms currently being developed on Wall Street." In their paper, Almgren and Chriss use the mathematics of the calculus of variations to pose optimal trading as an optimization problem that can be solved once the cost of transacting is known. In practice, knowing that cost requires a great deal of detailed data analysis to succeed. It is not an exaggeration to state that this entire line of business would not be possible without mathematics or without mathematically sophisticated people to research it and operate it.

While the idea of quantitative asset management was certainly around in 1996 when the first financial mathematics programs were being formed, it was by no means as big or important in the asset management industry as it is currently.

6 See http://www.turtletrader.com/trader-simons.html.

7 Data from the Hedge Advisory Group Weekly Scoreboard, February 10, 2005.

8 See http://www.deshaw.com.

9 See "The Orders of Battle," 38 Institutional Investor 77 (November 2004).

Today, hundreds of funds offer products specializing in quantitative trading. On the other hand, in 1996, the notion of algorithmic trading, which is becoming a key offering of Wall Street Brokerage Houses, was not yet on the horizon. This, in fact, has been the trend. As the importance of mathematics becomes recognized in one area of finance, it finds its way into another, and another. As more Wall Street houses, hedge funds, and money managers realize profits through the use of quantitative methods, new and different business centers will find ways to use quantitative methods to make money. And all along the way, demand for mathematical finance graduates will grow.

THE FUTURE

Programs in mathematical finance are nearing their ten-year anniversary, and the big question is their future. It is never easy to tell, but in the ten years, several clear trends have emerged. First, financial derivatives and the demand they create for quantitatively talented individuals are likely to be a permanent part of the financial landscape. Derivatives themselves afford the U.S. and global economy so much flexibility in terms of risk mitigation that it is hard to imagine what could drive them away. Noteworthy is that despite a long list of financial disasters pinned squarely on derivatives (Orange County, Long-Term Capital Management, and the collapse of Barings, to name a few), the long-term fallout has been zero.

Second, quantitative methods applied to money management have been a growing part of the money management and hedge fund landscape. High-profile success stories such as Renaissance Technologies and D.E. Shaw have made rocket scientists an increasingly accepted part of the business.

Third, new businesses are increasingly made possible by high-powered technology and high-powered mathematics. Algorithmic trading is an entirely new business that would not be possible without sophisticated mathematics.

ANOTHER HUGE AREA WHERE MATHEMATICS AND TRADING ARE COLLIDING IS IN WALL STREET'S BIGGEST NEW TRADING BUSINESS, ALGORITHMIC TRADING.

All of these trends point to one undeniable truth. In the future, Financial Mathematics graduates will be in demand. Another less-talked-about trend is also having an influence. The reality is that, when financial mathematics programs were first introduced, a great many Ph.Ds in physics and mathematics were flooding into Wall Street. These students were from a by-gone era when, if you were smart in math or physics, you usually went into teaching or research. But through the 1990s, job opportunities in academia became increasingly less abundant, while at the same time, job opportunities on Wall Street became increasingly more available. The result was a kind of mass exodus from the junior ranks of academia. In the end, it is much more likely that a significant portion of would-be academics will skip the long, tortuous route of becoming an academic, only to leave for Wall Street, and instead go directly to a fast-track way to Wall Street— financial mathematics. This bodes well for financial mathematics graduates.

When financial mathematics programs first appeared, they faced many obstacles, chiefly a feeling among hiring managers on Wall Street that they were superfluous. "Why should we hire you when we can hire any of a large number of better-trained Ph.Ds?" That may have been true in 1996, but it is becoming less so now because as job opportunities on Wall Street for quantitative analysts have become more well known, many potential Ph.D students have decided to drop out *before* ever going into the programs. They see financial mathematics programs as a short cut to Wall Street. The result will likely be fewer physics and math Ph.Ds available for Wall Street in the future. Put a different way, the future is very bright indeed for financial mathematics.

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[10] See "Optimal Execution of Portfolio Transaction," 3 Journal of Risk 5 (Winter 2000).