Assisting Defined-Benefit Pension Funds: An Application of Longevity Bond and Glide Path Strategy in Asset Allocation

Abstract

In this study, we tried mitigate the potential crisis of the US economy by identifying the major reasons for the shortfall of the Defined-Benefits plan. We described a simplified asset allocation model for a specific cohort of people, which takes care of longevity risk and salary growth rate risk. We can add more factors but the basic idea behind the model will remain the same. The model introduces glide path strategy and longevity bonds in our asset allocation. We show that pension fund asset reallocation reduces the shortfall risk and reduces the costs for companies.

Keywords

Shortfall Problem, Mortality Model, Longevity Bond, Glide Path Strategy, Simulation

1 Introduction

Defined-Benefit (DB) retirement plan is an employer sponsored plan, which defines the mathematical benefit an employee will receive at retirement. The higher your ending salary, the longer you stay with the employer, and the older you are when you retire will result in a higher pension. Thus, pension plans make very good sense for employees who stay with one employer most of their working career. Although, the money placed into a pension plan is managed by the employer, it is not an asset of the corporation and belongs to the employee participants of the plan. Unlike hedge funds, which are too ambitious about earning high profits, the primary objective of a benefit plan is to meet its obligations to provide promised retirement benefits to the employees and this process is called Asset/Liability Management (ALM). The fund works fine until the assets in the balance sheet are equal or greater than the liabilities. Falling interest rates, rising human longevity, and wavering performance of equity markets has resulted in making liabilities greater than assets, and now the shortfall risk has become a national issue. The problem, which many companies want to solve, is the ways in which they can de-risk and decrease the underfunded rate of DB plan.

There are many factors which contribute to the above problem: 1) mortality rate, 2) salary growth rate, 3) interest rates and 4) asset portfolio. If we fix everything else except for asset portfolio, our only consideration will be to pick the right asset portfolio to meet certain shortfall gap. However, the challenge of this problem is that there are many risks and uncertainties. The horizon of DB plan is usually over 30 years and it is because of this long horizon that it’s hard to estimate the age, at which someone will die, the size of benefits and contributions, the interest rate and the inflation rate. Although we have established that there are many risks and uncertainties, the most important ones are: 1) longevity risk, and 2) salary growth rate.

Usually, pension plans take all the employees into consideration. In order to simplify the problem, we decided to break the employees into several cohorts based on their age, and focus only on one cohort of employees. If we can solve the shortfall problem of one cohort of employees, then we can apply similar strategy for other cohorts as well. Our goal is to solve the shortfall problem, and match the liabilities and assets after the cohort of employees retire from the company. To solve our problem statement, we have built a model that mimics the main factors of the DB plan. Since we don't have access to the real world data, we have applied simulation to test our model.

We will explain our solution in five different parts: introduction, liability valuation, asset allocation strategy, simulation, and conclusion. The first part is a brief introduction of the shortfall problem of DB plan. In part two, liability valuation, we will first introduce the Lee-Carter mortality model, and then apply it to the calculation of benefits and liabilities. In part three, the asset allocation strategy, we will introduce a glide path strategy for equity and inflation-indexed bond, and include longevity bond in the asset allocation to reduce the longevity risk. In part four we will show and explain our simulation results, and part five is a brief conclusion of our solution to the shortfall problem of the DB plan.

2 Liability Valuation

In this part, we will first introduce the Lee-Carter mortality model, and then apply it to the calculation of benefits and liabilities

2.1 Mortality model

We embed the longevity risk into the scenarios by deliberately choosing a mortality model. The model we are going to use must capture the fact that the average life expectancy of a cohort will increase gradually with time. With this concern, we find that the Lee-Carter Mortality Model should be a qualified one. We choose this model for several reasons. Firstly, the stochastic model explains the longevity phenomenon nicely without adding too much of mathematical complexity. Secondly, Lee-Carter Model and its derivative versions have been widely applied to many practical business scenarios. The model is as follows:

$$\ln M_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \tag{1}$$

where $M(x,t)$ is the mortality rates for an age group x and $\varepsilon_{x,t}$ is an standard Guassian r.v..

$$ln\ (M_{x,t}) = a + bk(t) + \varepsilon_{x,t} \tag{2}$$

We use a random walk process to simulate the evolution of $k(t)$.

$$k_t = k_{t-1} + \alpha + \sigma\varepsilon_t \tag{3}$$

where α is the shift of the random walk and ε_t is an standard Guassian r.v..

The randomness comes from $\varepsilon_{x,t}$ and ε_t, whose correlations can be fitted by historical data.

One shortfall of the model comes from the variance of k_t, which grows unlimitedly with time and leads to an implausible result: life expectancy of people will increase to an incredible number when the forecast length is too long. However, this issue is not very problematic in our case for we do not need to forecast a very long period of time.

Based on eq.(2) and eq.(3), we can easily derive the surviving rate of the cohort at time t as follows

$$q_{x,t} = \prod_{i=1}^{t}(1 - M_{x+i,i}) \tag{4}$$

Obviously the best forecast of m_i should be

$$\widehat{M}_{x+i,i} = a_{x+i} + b_{x+i}\hat{k}_i \tag{5}$$

where, $\hat{k}_i|\mathcal{F}_{i-1} = k_{i-1} + \alpha$

We use mortality data of US from 1945-2012 as regressive input and get the least-square fits parameter estimates.

2.2 Benefit and liability estimation

Once we have the mortality model, we can compute the benefit (B) each year, and calculate the total liabilities as the sum of all unpaid discounted benefits. Mortality rate (M) is derived from Lee-Carter Mortality model. The number of employees alive (N) is calculated according to the mortality rate (eq. 6). We assume the average initial benefit per employee is 1.5% of average ending salary multiplied by the number of years service (eq. 7), and the liability is the sum of all unpaid discounted benefits (eq. 8).

$$N_t = N_{t-1} * (1 - M_t) \tag{6}$$

$$B = \overline{Ending\ salary} * No.of\ years\ service * 1.5\% \tag{7}$$

$$L_t = \sum_{\tau=t} Discounted\ B_\tau * N_t \tag{8}$$

3 Asset Allocation Strategy

The general equilibrium function of our DB plan asset is as follows:

$$\sum_i \omega_{i,t} A_t = \sum_i \omega_{i,t-1} A_{t-1}(1 + R_{i,t}) + LB_t + C_t - B_t \tag{9}$$

where,

$\sum_i \omega_{i,t} A_t$ is the total asset. $\omega_{i,t}$ is the weight of asset $A_{i,t}$ at time t.
$R_{i,t}$ is the return rate of asset $A_{i,t}$ at time t.
LB_t is the cash flow of longevity bond at time t.
C_t is the contribution to the pension fund at time t.
B_t is the benefit paid to the employees at time t.

3.1 Glide Path Strategy

Glide path strategy is that, as funded status rises, we gradually decrease our allocation in equity and increase allocation in bonds to match the assets and the liabilities. We mainly invest our Total Assets (TA) in three categories: Equity (E), Inflation-Indexed Bond (IIB) and Longevity Bond (LB). We will apply the glide path strategy to the allocation of equity and inflation-indexed bond. The weight of equity and inflation-indexed bond is based on the Funded Status (FS) in the previous year, while, due to the specialty of the property of longevity bond, the allocation of longevity bond is considered separately from the other two assets. We will introduce the longevity bond in detail in the next section.

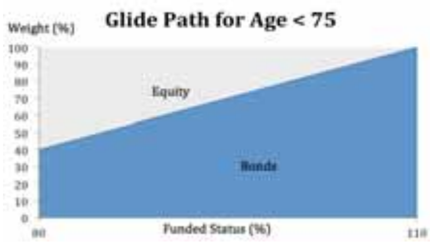
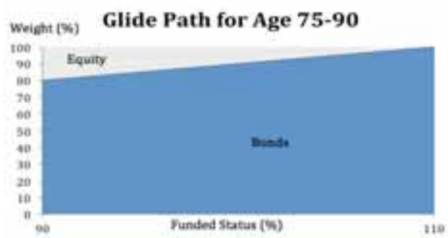
$$TA_t = \omega_{E,t} A_{E,t} + \omega_{IIB,t} A_{IIB,t} + LB_t \tag{10}$$

Funded status is defined as the total asset divided by the total liability:

$$FS_t = TA_t / TL_t \tag{11}$$

The allocation of the equity and inflation-indexed bond at different times is shown in table 1. In our strategy, for employees younger than 75 years old, we decrease the weight of equity from 60% to 0% as the funded status increases from 80% to 110%. For employees between 75 and 90 years old, we decrease the weight of equity from 20% to 0% as the funded status increases from 90% to 110%. After 90 years old, we allocate no equity in their portfolio.

Table 1: Allocation of the equity and inflation-indexed bond at different time

25<t<75	75<t<90	t>90
$\omega_{E,t} = \begin{cases} 0.6 & if\ FL_{t-1}\epsilon(0, 0.8) \\ -2 * FL_{t-1} + 2.2 & if\ FL_{t-1}\epsilon(0.8,1.1) \\ 0 & if\ FL_{t-1}\epsilon(1.1, \infty) \end{cases}$	$\omega_{E,t} = \begin{cases} 0.2 & if\ FL_{t-1}\epsilon(0,0.9) \\ -FL_{t-1} + 1.1 & if\ FL_{t-1}\epsilon(0.9,1.1) \\ 0 & if\ FL_{t-1}\epsilon(1.1, \infty) \end{cases}$	$\omega_{E,t} = 0$
		

3.2 Longevity bond

The purpose of introducing longevity bond is to hedge longevity risk, which is the risk that certain cohorts live longer on average than expected. Longevity risk can cause many problems in defined benefit plans' design and pricing. Individual risks such as unexpected death are idiosyncratic and therefore can be diversified; aggregate risks such as longer average death via medical improvements are systematic which is hard to be diversified by existing financial derivatives. The first longevity bond came out in 2004 by the European Investment Bank. The total face value of the bond was $540 million with the expiration time of 25 years and the annual coupon was $50 million multiplied by real survivor rate. It was a breakthrough attempt to solve longevity problems at that time, but after entering into 2010s, some new problems arose and they need new solutions. Baby-boomers continuously pass over retiring threshold, posing potential asset-liability imbalances among existing pension plans. Increasing interest rates risen discount rate that reduces present value of liabilities while S&P 500 index was up in two consecutive years, improving asset exposure that bridged the gap between asset and liability again. These factors combined together created many uncertain factors that need to be addressed quickly using financial engineering perspective both innovatively and efficiently. This part will introduce some new features of the newly designed longevity bond that solves some of the problems above.

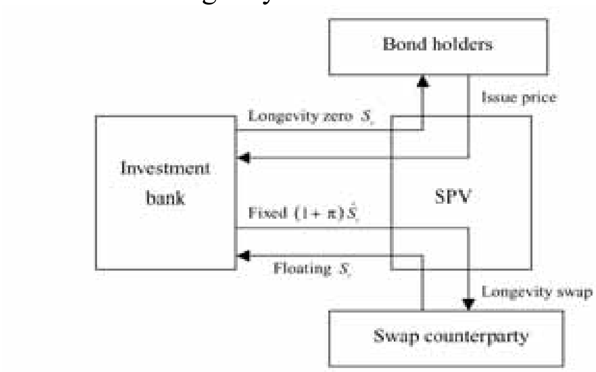
Mortality securization is the key idea (shown in figure 1). One basic approach to decompose cash flow is proposed by Blake, Cairns, Dowd and MacMinn (2006). The bond is issued by the Investment Bank (IB), which acts like entering into two swaps with two counterparties. The first swap is with Bondholder. Bondholder pays a lump sum issue price to IB, which pays floating amount S_t to Bond holder each year depending on the mortality index I_0. Mortality risk has the following equation:

$$I_t = \Sigma K_{i,t}(G^m M_{i,t}^m + G^f M_{i,t}^f) \quad (12)$$

where G represents gender information, M is mortality rate for age group and K is the weight for this age cohort. The index tends to rise as mortality rate and weight of this group increases.

In the second swap, The IB pays a fixed amount $(1+\pi)\hat{S}_t$, where π is the premium that ensures the zero initial value of the swap. In the perspective of bondholder, he will receive S_t every year after offsetting cash flows. In the perspective of IB, the credit risk now becomes the fixed leg only. Also, Special Purpose Vehicle plays a crucial role in assuring credit of both parties.

Figure 1: Cash flows of a longevity bond



There are a couple of characteristics to mention before pricing this bond. Since this bond is closely related to mortality risk, the payment should vary with the change of mortality rates measured by I_0. Specifically, when ignoring other risk factors:

$$p_t = \frac{Max(UI - I_t) - Max(LI - I_t)}{UI - LI} * \text{coupon payment} \quad (13)$$

where p_t is payoff at time t, UI and LI are upper and lower bounds of a certain mortality thresholds and it is mortality at time t.

This equation implies that payoff is related to the relationship of current mortality index and threshold I_0. It is easy to see that payment reduces linearly when I_t increases during a certain range. Specifically, when it is below lower boundary, the payment will not be discounted. When it is between upper and lower bound, it is a linear reduction as mortality goes up. As mortality rate goes above upper bound, then payment is fixed again but at a lower level. A good way to think about the mechanism is that when mortality is high, there will be fewer funds available for retirees, which means fewer payments are needed for retiring funds.

In the real world, however, there are some other risks to consider. As the case problem mentions, interest rate has risen so liabilities are reduced because of the discounting rate. S&P 500 was up almost 30% and in 2014 it was up another 12%, which improves funding ratio through equity exposure. To hedge such changes and therefore keep the correlation between assets and liabilities, the payment should also reflect the interest rate and asset value changes. So the final payoff is:

$$p_t = \frac{Max(UI - I_t) - Max(LI - I_t)}{UI - LI} * e^{r_t - r_{t-1}} * \left(1 + \ln\left(\frac{SP_t}{SP_{t-1}}\right)\right) * \text{coupon payment} \quad (14)$$

where r_t is the risk-free rate at time t and SP_t is the S&P 500 stock index at time t. Intuitively, by adding these two terms, the impact of interest rate and stock index movement will be eliminated. This is because when r and SP increases, P_t and funding ratio will increase so that more payments can be made to retirees. One thing to notice here: natural and log terms are used so that changing effects will be controlled to a certain degree that reduces likelihood of large volatility in the payment also asset-liability matching.

We involved the existing longevity bond sold by European Investment Bank in our asset portfolio. We have applied a dynamic strategy with three triggers at different time for longevity bond allocation. The first time is when the cohort of employees reaches their retirement age and the asset is at least 7 times larger than the benefit of that year, we spend 10% of the asset on longevity bond to cover part of the longevity risk; the second trigger is when the cohort of employees reaches 90 and if the asset is larger than liability, we spend 70% of the excess funds on longevity bond to match the asset and liability; the third trigger is when the age of the cohort is beyond 98, we spend 40% of our asset on the longevity bond, and the reason for the third trigger is that, at this stage, the amount of benefits are relatively smaller because very few employees are still alive at this time. Therefore, we can spend a larger portion of the assets on longevity bond and let the longevity bond payment cover most of the benefits of the remaining several years. The purpose of this dynamic strategy for longevity bond is further explained in the simulation part along with the results.

4 Simulation

In this part, we are going to apply the simulation to test our asset allocation strategy. Based on our analysis on the liability valuation part and asset allocation part, we will use mortality model to estimate total benefit and liabilities of the defined-benefit plan, and involve longevity bond and glide path strategy in our asset allocation strategy.

4.1 Assumptions

Before we analyze the simulation of our asset allocation strategy for defined-benefit pension plan, we need to specify all assumptions in our model:

1. We consider only one cohort of the employees. There are 100,000 employees in total at the beginning. All the employees are 25 years old at t = 25, and all employees retire at 65 years old at t = 65. No employee resigns.
2. Mortality rate (M_t) is derived from the Lee-Carter Mortality model (eq. 2 and eq. 3). The number of employees alive (N_t) is calculated according to the Lee-Carter Mortality model.

$$N_t = N_{t-1} * (1 - M_t)$$

3. Salary (S). The base salary at t = 25 is \$60,000 per person. The growth rate of salary each year is normally distributed, with a mean of 0.09, and standard deviation of 0.01.

$$S_t = S_{t-1} * (1 + 0.09 + 0.01\varepsilon_t) \qquad \varepsilon_t \sim N(0,1)$$

4. Benefits (B). The first benefit (B_{66}) per employee is 1.5% of the last year's salary multiplied by the number of years each employee has worked (eq.7), and the benefits in the following years are adjusted from the inflation rate (inf).

$$B_{66} = S_{65} * 40 * 1.5\%$$

$$B_t = B_{t-1} * (1 + inf_t)$$

5. The inflation rate and discount rate (D) is normally distributed as follows:

$$inf_t \sim N(0.02, 0.00005)$$

$$D_t = D_{t-1} + 0.0001 + 0.0025\varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

6. Liability (L) is the sum of all discounted benefits (eq. 8).

$$L_t = \sum_{\tau=t} Discounted\ B_\tau * N_\tau$$

7. Contribution in year t is 10% of the total salary of all employees in year t.

$$C_t = S_t * N_t * 10\%$$

8. The return rate of the inflation-indexed bond is the same as the inflation rate of the same year, while the "price path" of equity (E) follows the following equation, with a drift of 0.08 and a consistent volatility of 0.1.

$$\frac{dE_t}{E_t} = 0.08\,dt + 0.1\,dW_t \qquad dW_t \sim N(0, dt)$$

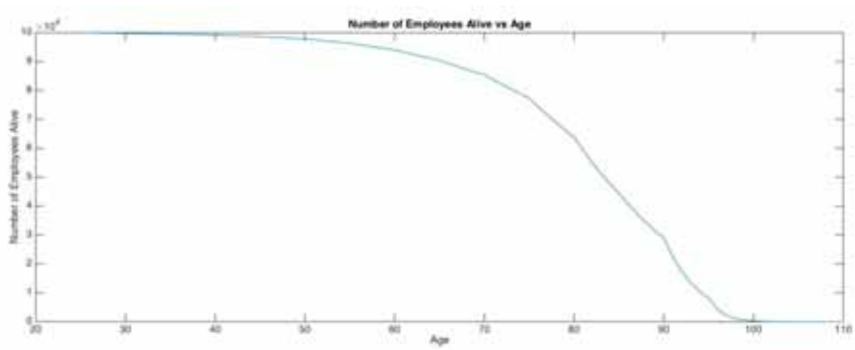
9. Longevity Bond. We use an existing longevity bond introduced in part three. Its maturity is 25 years and the price of one unit longevity bond is $540 million. The payments of the longevity bond are based on the mortality index I_t (calculated in eq. 13) and are calculated (eq. 13) as follows:

$$P_t = \frac{\mathrm{Max}(UI - I_t) - \mathrm{Max}(LI - I_t)}{UI - LI} * \$50\ \text{million}$$

4.2 Simulation Results

We simulated our proposed model on a cohort of 100,000 employees with a 1:1 ratio between men and women. Further, we assumed that this cohort of employees has an age of 25. For our mortality model we borrowed data from the paper "Modeling and Forecasting U.S. Mortality"[1] who used historical data from 1933-1987 in order to simulate their results. An example run for our mortality model is given in figure 2.

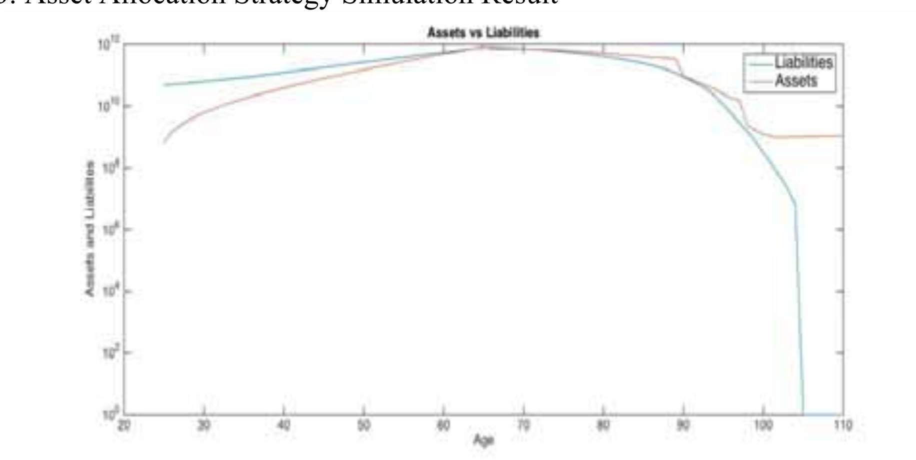
Figure 2: Mortality Simulation



We can see from the figure 2 that the initial mortality rate is pretty low but eventually it starts to increase at a fast rate and almost all employees die by the age of 105. Based on this model it is a rare scenario where any employees are alive beyond the age of 106. We also plotted the resulting graph for assets and liabilities which is given in figure 3.

From simulation result (figure 3) we can see that the fund starts with a relatively low funding level; therefore, the growth rate of assets is high because of a high share of equity (60%). Based on the glide path strategy the growth rate is expected to slow down as funding level increases because the plan reallocates assets and increases the percentage of bonds held. At age 66 the fund the fund buys the first set of longevity bonds to cover part of the longevity risk. This reduces the assets held and since the share of equity is much smaller by this time we notice a relatively smooth period with a close match between assets and liabilities. At age 90 the fund again reduces the assets held by buying another set of longevity bonds based on the difference between assets and liabilities. This step as explained in the strategy part is a step towards stability because the fund is meeting its requirements and stability is more important than growth by this point. Finally, at age 98 the fund buys a final set of longevity bonds. By this point only a small population of employees is alive and can be covered by liquidating majority of the assets into longevity bonds.

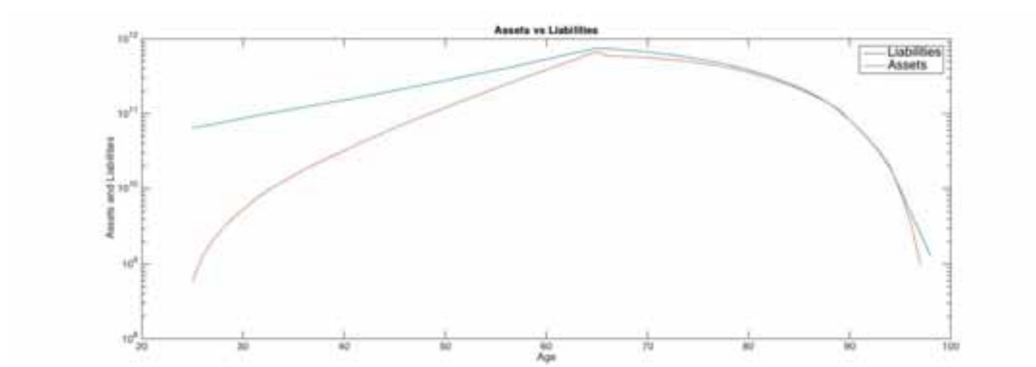
Figure 3: Asset Allocation Strategy Simulation Result



We ran our simulations several times and noticed two key statistics:

1. The fund always runs successfully and matches its liabilities till employees are aged 90. Beyond age 90 the fund has a success rate of approximately 70%. The reason for failure (a failure example shown in figure 4) is usually a poorly performing equity market. This issue can be better resolved practically rather than a simulation because a fund manager would be able to choose better equity in case of poor performance while keeping the allocation between equity and bonds as per glide path. This feature is not possible in a simulation because the simulation assumes the stock returns follows a stochastic process and does not reallocate in case of poor equity performance.

Figure 4: Asset Allocation Strategy Simulation Result (Failure)



2. The fund cannot have zero assets at the end of its life. This is because the last benefits payment is usually of the order of millions and the fund needs to make sure that they have this money. Any attempt to precisely match the liabilities increases the failure rate in the final years of life. Therefore, our strategy makes sure that we buy enough longevity bonds to last us through at least till the unlikely scenario of employees living until age 110. In majority of our simulations all employees die by the age of 106. Therefore, we are left with some amount of assets in the final 4 years.

5 Conclusion

We can say that the DB shortfall is a result of many factors. Though, we built a simplified model for a specific cohort of people by taking care of only two factors namely longevity risk and salary growth rate risk and by introducing the glide path strategy and longevity bond in our asset allocation. We can easily say that our strategy worked based on the simulation results. We can also apply similar strategy for other cohort of employees. In real world, it would be even more difficult to solve the shortfall problem because we need to take into consideration a lot of other factors (early retirement, bearish stock market and medical improvement which lead to more serious longevity risk etc.) which we ignored in our model. Despite all the risks, shortfall problem can easily be solved by simply increasing the contributions. However, we will have to think of better strategies for pension fund asset allocation in order to reduce the pension risk and the cost for companies.

6 Notations

Symbol	Meaning
$A_{i,t}$	Allocation of asset i at time t.
B_t	Benefit per employee at time t.
C_t	Contribution at time t.
D_t	Discount rate at time t.
E	Equity.
FS_t	Funded status at time t.
G	Gender weight in the mortality index.
I_t	Mortality index in longevity bound.
inf_t	Inflation rate at time t.
LB_t	Cash flow of longevity bond at time t.
$M_{x,t}$	Mortality rate of cohort x at time t.
N_t	Number of employees alive at time t.
p_t	Payment of longevity bond.
$q_{x,t}$	Survive rate of cohort x at time t.
$R_{i,t}$	Return rate of asset i at time t.
S_t	Salary per employee at time t.
SP	S&P 500 stock index.
TA_t, TL_t	Total asset and total liability at time t.
UI, LI	Upper bound and Lower bound of mortality index.
$\omega_{i,t}$	Weight of asset i at time t.

Reference

John M. Mulvey, Koray D. Simse and Zhuojuan Zhang (2008). *Assisting Defined-Benefit Pension Plans*, Operations Research, Vol. 56, No. 5, pp. 1066–1078, issn 0030-364X, issn 1526-5463,08,5605,1066.

Shumin Chen and Zhifeng Hao (2013). *Funding and Investment Decisions in a Stochastic Defined Benefit Pension Plan with Regime Switching*, Lithuanian Mathematical Journal, Vol. 53, No. 2, pp. 161–180

Ronald D. Lee; Lawrence R. Carter (1992). *Modeling and Forecasting U.S. Mortality*, Journal of the American Statistical Association, Vol.87, No.419, pp. 659-671.

David Blake, Andrew Cairns, Kevin Dowd and Richard MacMinn (2006). *Longevity Bonds: Financial Engineering Valuation, and Hedging*. The Journal of Risk and Insurance, Vol. 73, No. 4, 647-672

Samuel H. Cox, Yijia Lin, Ruilin Tian and Jifeng Yu (2013). *Managing Capital Market and Longevity Risks in a Defined Benefit Pension Plan*, the Journal of Risk and Insurance, Vol. 80, No. 3, 585–619 DOI: 10.1111/j.1539-6975.2012.01508.x

Wai-Sum Chan (2013). *Longevity Risk: Recent Developments and Actuarial Implications*, Society of Actuaries, The Chinese University of Hong Kong (A SOA Center of Actuarial Excellence).

Ricardo Josa-Fombellida and Juan Pablo Rincon-Zapatero (2008). *Optimal Asset Allocation For Aggregated Defined Benefit Pension Funds with Stochastic Interest Rates*, Universidad Carlos DE Madrid, Working Paper 07-81, Economic Series 48.

Daniel Bauer, Jochen Ruß (2006), *Pricing Longevity Bonds Using Implied Survival Probabilities*.

Con Keating (2003). *Pensions Modeling*, IMA Journal of Management Mathematics, 2003.

Jeffrey R. Kamenir (2009). *How to Make Defined Benefit Pension Plans Attractive to 21st Century Employers*, Benefits Quarterly, Second Quarter 2009, page 51-56