CDS on CDO Portfolio A Fundamental Factors' Model for Hedging, Collateralization and Capital Reserves

TIME SQUARED EXPLORERS

Abstract
In this study, we have identified the cause of AIG's failure in 2008 to be large unhedged CDS exposures on CDO portfolios. We built a fundamental factors model to price CDS-CDOs as well as create hedging strategies. We specified subprime mortgage default and prepayment hazard rates to be dependent on common fundamental factors including interest rates and housing prices as well as deal specific idiosyncratic factors. We calibrated our base fundamental factor parameters with loan level ARM mortgages and make certain assumptions for the CDOs. We then studied the hedging performance with instruments like ABX.HE indicies, Euro dollar futures and Home Price Index futures. We propose a new collateral scheme based on CVA values of both counter- parties. We also demonstrate a framework to calculate the VaR on the CVA and expected loss distributions of the CDS after col-lateralization. We then propose a method to determine the capital reserve needed for CDS contracts based on two VaRs.

Keywords
Fundamental Factors, Hedging, Collateralization, Capital Reserve, CDS-CDOs

Contents

1. Introduction

The near collapse of the American International Group (AIG), the largest insurance company by then, is one of the most severe events during the subprime-mortgage crisis. The bailout of AIG, along with other concurrent collapses of some major financial institutions, led both the practitioners and regulators to reflect upon their legacy risk management practices. Under the context of emphasis on deregulation and financial innovations after Financial Services Modernization Act of 1999 (FSMA) and the Commodity Futures Modernization Act of 2000 (CFMA), disputes had arisen over how to strike the balance between the cost of carrying stringent risk management and regulatory burden and the benefits from rolling out structured financial instruments in a more inter-related financial system. Thereafter, we saw a market inclination towards vanilla instruments. The new regulatory frameworks, Dodd Frank Act and BASEL III accord (Basel Committee, 2010), both aim at ensuring the wellness of financial system.

In this study, we will revisit bailout of AIG, based on which we will propose a fundamental factor approach to mitigate the risks of then AIG's most problematic position - the credit default swap portfolio on collateralized debt obligations. As we shall see later, our factor approach will consistently offer three layers defense - hedging, collateral management and capital reserve - each of which will address the an particular problem of AIG's collapse and roots deeply in the economics. The rest of the report is organized as follows

2. A Reflection on How AIG Collapse

2.1 AIG Bail-out Recap

AIG's fall epitomizes the aforementioned issue and the ongoing debate on managing the complicated financial products. It is generally agreed that the AIG's credit default swap portfolio on super-senior tranches of multi-sector collateralized debt obligations (CDS-CDOs) is the fatal cause of AIG's "death" (Sjostrom Jr, 2009; Boyd, 2011). The disruption in the housing market in 2007 seriously devalued the mortgages and mortgaged-based derivatives (American International Group, 2007, 2008). Through both direct investments in RMBS and the CDS-CDOs position, AIG maintained a gigantic exposure to mortgage market and hence had to record billions of realized and unrealized losses. Questioning the AIG's ability to fulfill the obligations, counter-parties to AIG initiated collateral calls one after another, resulting demand of liquid assets amounting to more than 50 billions - a disastrous number that pushed AIG towards the verge of bankruptcy which was avoided only after the Federal Reserve put together $85 billion credit facility to bail out AIG (Sjostrom Jr, 2009)

However, it is inaccurate to solely attribute the consequence to CDS-CDOs or the abrupt decline of housing market. The design of the CDS aims to benefit from risk diversification and risk transferring through securitization (Lucas et al., 2007). CDS arose from the need of credit risk protection. Both of these instruments, in theory, are to achieve better risk allocation among players in financial market. As long as the risks are priced appropriately to compensate the firms who bear the risk, nothing is economically wrong with the CDS-CDOs itself. Hence, AIG fell not because they maintained a CDS-CDOs portfolio on their book but rather they took on risks which exceed their capabilities to manage effectively:

2.2 Cause I: Gigantic Unhedged Exposure

Firstly, the most identifiable problem is AIG's gigantic exposure to the mortgage market through both direct investments and the unfunded CDS portfolios. Since AIG dabbled the CDS business in 1998 through its subsidiary called AIG Financial Products (AIGFP), AIG gradually became the major protection seller in the market, mainly due to AIGFP's AAA credit ratings inherited from AIG via the contractual guarantee agreement (TPM, 2009). Such a AAA-rating business model enabled AIGFP to price its CDS competitively and accumulate market share quickly.

Throughout early 2000s when market was deregulated and financial innovations were encouraged, AIGFP had expanded its CDS coverage extensively from conventional corporate bonds to more exotic OTC contracts on CDO. According to American International Group (2007), by the end of 2007, AIGFPs CDS position was as large as $527 billion, which was quickly increased from $203 billion in 2003. Among the $527 billion, $378 billion was sold to mainly European banks with the purpose to reduce the capital requirements ("Regulatory Capital Relief") and the $149 billion, including the toxic $78 billion protection on CDOs, was contracted with the clients, such as Goldman Sachs, who were interested then popular negative-basis arbitrage ("Arbitrage Portfolio") (American International Group, 2008).

Such a position, with hindsights, was too large given that AIG had only 2.3 billion in cash and $95 billion in equity to absorb the losses. The question on their capability to handle the big position was even more confounded as AIG directly invested in mortgage securities in its ordinary business and securities lending program (Sjostrom Jr, 2009). Yet, as written in AIG's 2007 Annual report, the management decided not to hedge their position in CDS with the preconception that super senior tranches are safe given their simulation and modified BET models. When the mortgage market did go against their bet, the loss was significant in a sense that CDS-CDOs solely contributed a $11.2 and $25.7 billion unrealized loss in 2007 and 2008, in addition to other realized impairment cost ($20 - $30 billion) in 2008 (American International Group, 2008).

However, had AIG intended to hedge the position some time before the financial crisis, situation would possibly not be more favorable to AIG. CDS are insurance-like contracts. As CDS is the most common and thus the cheapest sources of credit protection, it would be difficult to look for cheap alternatives to remove exposure from CDS. This is especially true when (1) the secondary market of both the reference asset and the CDS are not liquid enough, and (2) reinsurance market is not large. To cut or hedge billions of exposure in any market during distressed time was intrinsically hard, not to mention their contracts are mostly OTC. Hence, perhaps, AIG should have not put itself into a huge CDO pool at first.

2.3 Cause II: Problematic Credit Annex Support

The second problem, which is also the feature that differentiates the CDS-CDOs position from other direct mortgage investments, is the collateral posting obligations embedded to the CDS contracts. Such type of obligations imposes contingent demand on the firms liquidity and, in contrary to the asset devaluation, the inability to fulfill the cash obligation will lead to the immediate bankruptcy of the firm. After Goldman Sachs collateral calls as the CDO market worsened, major counterparties to AIG started following the practices, putting AIG into cash woes. The subsequent downgrade of AIG automatically applied another multiplicative factor to the already-huge collateral base. By September 2008, AIG was required to post almost $31 billion collateral solely against CDS on CDOs positions (American International Group, 2009). Adding the collateral from security lending business GIA, the number was as high as $54 billion - an amount was filled only after Fed stepped in (Sjostrom Jr, 2009; Boyd, 2011)

The collateral crisis on one hand was the result of management negligence but, fairly speaking, the terms of the Credit Support Annexes itself made collateral management difficult for AIG. According to AIG's FY2008 10-K, the calculation of collateral delivery amount against CDS-CDOs position is a two-step process

1. Determine the exposure: computed as the difference between the net notional amount and the *market value* of the underlying CDOs;
2. Determine the delivery amount: the collateral to be delivery equals the net exposure which is the exposure less the collateral which has already been posted.

In additional to the above framework, AIG is allowed to

- post no collateral when the underlying CDO loss is below certain threshold, typically 4% as those with Goldman Sachs.
- post less collateral with a multiplicative factor based on AIG's credit ratings.

There are at least 4 potential problems associated with the aforementioned CSA. Firstly, instead of using the replacement value of the contract, the collateral is "market-price based" in which the "market value of relevant CDO security is the price at which a marketplace participant would be willing to purchase such CDO security in a market transaction on such date". For illiquid reference asset, such as CDO security, whether the market quote is the accurate proxy of the true economic value of the asset is questionable. This is evident especially in distressed financial market. Stanton and Wallace (2011) shows that the ABX.HE index during the crisis was traded extremely low, implying an unrealistic high default and low recovery rate which have never been observed in Great Depression. Collateral mechanism based on these market information can be overly volatile due to the market sentiment. Moreover, collateral is to ensure the "payment of CDS when reference actually default" instead of "hedging the value of instrument". Marked-to-market at an extremely distressed market is overly stringent for AIG

Secondly, the mechanism of the collateral threshold and the dependency of the collateral amount on credit ratings directly resulted in the risk of "jump in liquidity needs". In other words, when the CDO market loss hit the threshold or credit ratings lowered, the collateral call can often be sudden and large. Moreover, the unexpected cash needs often coincides with financial distress which could lead to considerable difficulties in raising additional funds. Hence, such a jump can be detrimental as the cash-based obligation is probably one of the hardest types of obligation to fulfill and leads to immediate default no matter how much assets the firm has on book.

Thirdly, the trigger rule, despite it appears to be beneficial to AIG, tends to hide the liquidity risk and lead managers to overlook the collateral management practices. The supersenior tranches of CDOs were considered with really low default risk before the crisis. Hence the super-senior tranches were traded near par and AIG was not required to post collateral as CSA was not triggered. This induced the management to accumulate large position while being unaware of the liquidity exposure, in that their position was almost free of cash burden. However, when the underlying asset goes bad, as in the case of AIG, the liquidity risk can be too large to be managed.

Lastly, the CSA design is loosely related to the credit risk of both counter-parties and the variability of the underlying. On one hand, the determination of the collateral trigger appears arbitrary and rigid over the life of the CDS contract. On the other hand, the infrequent adjustment of the credit rating means the counter-party risks are not captured timely. Both of the aforementioned disadvantages suggest the lack of economic interpretation of this CSA settings, veil the true nature of counter-party risk and hence make the collateral hard to manage.

2.4 Moral Hazard of the Bailout

The FRBNY thought it would be unethical to leverage this threat of bankruptcy, since it knew it was not willing to let AIG default. By imposing onerous conditions to the $85 billion RCF, the Fed did a good job of minimizing the micro moral hazard in the short run. Unfortunately, this did signal that they were not willing to let AIG default, which ultimately led to the much larger moral hazard of bailing out the AIG counterparties.

As acknowledged by Fed Vice Chairman Donald Kohn at a Senate Banking Committee hearing, the aid to these counterparties contributed to moral hazard and "will reduce their incentive to be careful in the future". The fact that the counterparties did not take a haircut has been widely criticized, including by Neil Barofsky, the former inspector general of the Troubled Asset Relief Program (TARP). Barofsky told congress that "No lessons were learned from the counterparties, other than, if you do business with a giant, too-big-to-fail institution, you don't need to worry about it because Uncle Sam is going to sit there and backstop all of your bad bets."[1] This could led to a market distortion where investors, creditors and counterparties don't bother with due diligence of these large, interconnected financial institutions, which in turn could lead to lower borrowing costs for these financial institutions. According to Senator Elizabeth Warren[2], the Too-Big-to-Fail problem has in fact gotten worse since the financial crisis. In a speech promoting the 21st Century Glass-Steagall Act, she said

> *Today, the four biggest banks are 30% larger than they were five years ago. And the five largest banks now hold more than half of the total banking assets in the country. One study earlier this year showed that the Too-Big-to-Fail status is giving the 10 biggest U.S. banks an annual taxpayer subsidy of $83 billion.*

[1] Quote retrieved from http://hereandnow.wbur.org/2013/09/13/tarp-watchdog-banks

[2] Quote Retrieved from https://www.commondreams.org/headline/2013/11/12-11

3. Fundamental Factor Model

The aforementioned analysis of fall of AIG's suggests that at least three lines of defense can be establish to mitigate, if not avoid, the downside risk of huge mortgage position. The firm should (1) reduce the exposure by either trimming the position or hedging the losses, which could avoid the abrupt loss of credibility and hence maintain the funding capabilities in a disrupted market; (2) establish a better collateral agreement that will facilitate a better understanding of credit exposure and collateral management, and (3) set aside adequate amount of firm-wide capital to serve as the last defense should the above trade-specific risk management fails.

However, as the protection seller of bond, looking for other alternatives to directly short the credit risk is difficult, as CDS is already the cheapest credit instrument. Hence, to avoid enormous hedging cost, especially when hedging large position, cross hedging tends to be a more feasible solution. Candidates of these instruments are the housing price index, ABX indices as well as the put options on firms which have exposure to mortgage market. These choices are based on the economic reasoning that the dynamics of these assets are more or less driving by the mortgage market.

3.1 Factors Specifications

This idea suffices the use of fundamental factor model instead of the common statistical model, such as copula or BET. The fundamental models can identify inherent risks and establish sound hedges. Specifically, we assume that the economy of mortgages are primary driven by housing price (housing index) and the interest rate. Mathematically, we adopted the log-normal model to capture the dynamics of the housing prices

$$dH_t = (r_t - q_t)H_t + \sigma^H dW_1 \tag{1}$$

and the interest rate r_t is given using the one-factor Hull-White model (Hull and White, 1993).

$$dr_t = \kappa(\theta_t - r_t)r_t + \sigma^R dW_2 \tag{2}$$

where q_t in equation (1) is the rental yield and assumed to be a constant $q_t = 0.25$ following Downing et al. (2005); W_1 and W_2 are Wiener processes under risk-neutral measure and are assumed to be independent for simplicity. That is, we let

$$dW_1 dW_2 = 0, \quad q_t = 0.25 \quad \forall t > 0 \tag{3}$$

3.2 From Factors to Cash Flows of Mortgages

Proportional Hazard Rate Model

It is known that the cash flows of mortgages and thereby mortgage-backed securities heavily depends on the prepayment risk and default probability. To model the influence of housing index and interest rate on mortgage payments, we adopted the proportional hazard rate model proposed by Schwartz and Torous (1989) in which (1) prepayment / default intensities are directly captured, and (2) the economic variables are explicitly incorporated. Mathematically, either the prepayment or the default hazard will follow

$$\lambda(t, X_i) = \lambda_0(t) \exp\{\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n\} \tag{4}$$

where $\lambda_0(t)$ is the baseline hazard controlling the shape of term structure, and the linear combination of covariates X_i's determines loan-specific shift of prepayment / default intensities. For the baseline hazard, we chose the log-logistic function

$$\lambda_0(t; \lambda, p, \gamma) = \frac{\lambda p (\lambda t)^{p-1}}{1 + (\lambda t)^p} \tag{5}$$

The major benefits of such a functional form is its flexibility in fitting a variety of shapes of prepayment or default pattern, such as the well-known burn-out phenomena of mortgages.

The selection of the covariates will determine how mortgages cash flows depend on the macro economy and the deal specifications. In this project, the following covariates are assumed in the proportional hazard rate models:

- **Coupon Differential**, which is defined as the difference between the coupon on the loan and the 3-month lagged 10-year interest rate. That is

  $$X_1(t) := \mathbf{CD}(t) := C - R^{(3)} \tag{6}$$

  Large coupon differential induce mortgage borrowers to prepay the loan and refinance the mortgage when interest rate is low;

- **Loan-to-Value Ratio**, which is defined as

  $$X_2(t) := \mathbf{LTV}(t) = L_t / V_t \tag{7}$$

  where L_t is the remaining principal of the loan at time t and V_t is the value of the property at time t. To model the dynamics of the property value, we further assume that the return of V_t coincides with the return of housing index

  $$\frac{dV_t}{V_t} = \frac{dH_t}{H_t} \tag{8}$$

The two covariates only explains the the impact of the macro-economic factors on mortgage cash flows. However, each loan pool can also be influenced by deal-specific variables, such as credit scores, property location etc. To account for the idiosyncratic components, we add a time-varying "factor" to proxy the residual information. Mathematically, the "third factor" takes the form

$$X_3(t) := \beta_0 + \varepsilon(t) \quad \varepsilon(t) \sim \text{i.i.d } N(0, \sigma^2) \tag{9}$$

and we further assume that the idiosyncratic factors are independent among deals. Combining the equation (6), (7), (9) with (4). Our modified hazard rate is

$$\lambda(t) = \lambda_0(t)\exp\{\beta_0 + \beta_1 \mathbf{CD}(t) + \beta_2 \mathbf{LTV}(t) + \varepsilon(t)\} \quad (10)$$

which can be obtained through simulation when parameter is calibrated.

Model Calibration

To calibrate the model, we used the mortgage loan-level data. With the hazard rate model above, we adopted the MLE to obtained the estimates.

Mortgage Cash Flows and Defaults

With the hazard rate term structure, $\lambda(t)$, the prepayment or the default probability in a given period is

$$PD(T_1,T_2) = \exp\left\{\int_{T_1}^{T_2} \lambda(t)dt\right\} \approx \exp\{\lambda(T_2)(T_2 - T_1)\} \quad (11)$$

where the last term approximates the probability with a piecewise constant hazard rate. Let $PD_P(t)$, $PD_D(t)$ denote the prepayment probability and the default probability respectively, then the mortgage payment can be computed through the following four steps (See Figure 12)

1. Given the last period ending balance $L(t-1)$, we can compute the average default amount of the mortgage pool with

$$\mathbf{Default}(t) = L(t-1) \times PD_D(t)$$

2. Then the amortization math can be applied to the remaining non-default balance of the bond to compute the principal and the interest portion of the payments. Denoted with $P(t)$ and $I(t)$ respectively.

3. After the amortization is determined, the prepayment is obtained by applying $PD_P(t)$ to the balance after the default and principal payment are deducted c

$$\mathbf{Prepayment}(t) = [L(t-1) - P(t) - \mathbf{Default}(t)] \times PD_P(t)$$

4. The ending balance is then

$$L(t) = L(t-1) - P(t) - \mathbf{Default}(t) - \mathbf{Prepayment}(t)$$

Iterate the above four steps until the mortgage pools terminate will gives us the default amount and the cash flows available for distribution for each period t.

3.3 From Mortgage Pool to MBS Cash Flows

Given the cash flows and the amount defaulted of the mortgage pool, we can determine the cash flow that each tranche receives based on the structure of the MBS. Here, for the simplicity of illustration, we assume that all MBSs have a simple waterfall structure *without* the accrued interest class (Z), overcollateralization (X) and the residual class (R) which are often observed in the real deals.

With the simple deal structure, as shown in Figure 12, cash flows are distributed according to the seniority of the tranches, with the most senior bond paid-off first. On the contrary, the default amount will cause the least senior tranche to be written-off first. Through this approach, the period-by-period cash flows and default amounts are obtained, based on which we can model the cash flow of CDO and other mortgage related securities

4. Cross Market Hedging of CDS-CDOs

4.1 Hedging with ABX.HE Indexes

In this study, we simulated 100 MBS deals by modeling at pool level the cash flows of defaults and prepayments. The MBS subordination structures are identical in those deals as they are in table 1. We randomly selected 100 tranches with 20 tranches from tranche 3 to 7 each and one tranche from each MBS deal to from a CDO. The CDO subordination structure is the same as the MBS deals. We performed the MC simulation with 2000 interests and HPI paths, and analyzed the joint distribution of the loss of the senior CDO tranche and the average loss of the MBS tranches. The average losses from the MBS tranches were calculated from randomly selected 20 deals. The CDO senior tranche incurred losses on 727 paths of the 2000 paths.

Table 1. Subordination Level of the Representative Deal

Tranche	Subordination (in %)	Spread (in bps)
1	22.1	18
2	18.4	26
3	13.5	33
4	11.5	38
5	9.8	41
6	8.2	47
7	6.7	80
8	5.3	100
9	4.2	200

From our simulation, we observed that the senior tranche of the CDO formed from equal numbers of tranche 3 to 7 of the MBS deals behaves generally in par with MBS tranche 3 and 4. The CDO senior tranche loss cash flows are very similar to those of the MBS tranche 3 and 4. They start to incur losses similarly. This is consistent with the diversification effect achieved with the CDO structuring. However, contrary to the intention of most CDO deals in the reality, we found the senior CDO tranche is inferior to the most senior tranche of the MBS deals. Our simulations results shows that the senior tranche of our CDO incurred about 50% loss before the most senior MBS tranche start to incur losses. This is likely due to the fact we calibrated parameters with subprime loans with high default rates, therefore the junior tranches has higher loss correlations, undermining the effect of diversification in the CDO deal.

In reality, a viable hedging strategy on the CDS on the senior tranche of CDO deal can be constructed as follows. First, the cash flows of the MBS deals are first modeled. This should be preferably done in the loan level, utilizing as much as loan level information as possible. Then the cash flows to the CDO (and thus CDS) and the 20 ABX deals are simulated under the same sets of scenarios. Their joint loss distributions are analyzed, and corresponding ABX class with the highest loss correlation should be chosen as the hedging instrument. In our simulation, the average of MBS tranche 4 and 5 are arguably the best hedging instruments, since they incur losses the same points with the senior CDO tranche and the expected losses have linear relationship when they start to occur.

4.2 Hedging with Interest Rate Futures and HPI Futures

Any financial instruments used to hedge large short CDS positions should have the following desirable features:

1. the instrument should have minimal counterparty risk so that the hedge does not fail in times of stress when large default payments need to be made. Given this consideration, we prefer exchange-traded instruments for hedging over over-the-counter instruments.

2. instruments used in dynamic trading strategies for hedging large positions should be highly liquid and have low bid-ask spreads so that transactions cost is not prohibitive, and market does not move against the trade. The inherent standardization in exchange-traded contracts lends itself to this consideration.

We describe a dynamic hedging strategy in which the hedge needs to be rebalanced every period. Interest rates affect CDS prices in 2 different ways: 1. cash flows are discounted by interest rates, and 2. default rate changes with interest rate due to which cash flows from the CDS change. However, for CDS contracts, the default effect is expected to dominate the discounting effect. Since ARM default rates increase as interest rates increase, we expect CDS value from seller's point of view to decrease when interest rate increases. Similarly, during a housing market boom, we expect default rate to decrease which would increase CDS value from seller's point of view.

To obtain interest-rate duration and housing price index (HPI) duration of the CDS contract, we run a regression of the form

$$CDS_i = \alpha + \beta \cdot r_i + \gamma \cdot HPI_i + \varepsilon_i$$

where i indexes Monte-Carlo paths, CDS_i represents CDS MTM along path i at time $t=1$, r_i represents interest rate along path i at time $t=1$, HPI_i represents housing price index value along path i at time $t=1$ and ε_i is the error term. To avoid in-sample fitting bias, we only use half of our Monte-Carlo paths to generate this regression fit. Given estimates for the parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$, our estimate for interest rate duration at time $t=0$ is $D^{ir}_{cds} = -\frac{\partial CDS}{\partial r} = -\hat{\beta}$ and for HPI duration at time $t=0$ is $D^{hpi}_{cds} = -\frac{\partial CDS}{\partial HPI} = -\hat{\gamma}$. Based on our simulation, we obtain $\hat{\alpha} = 4.67 * 10^6$, $\hat{\beta} = -4.29 * 10^8$, $\hat{\gamma} = 2.33 * 10^6$ and $R^2 = 58.69\%$. The high negative value of $\hat{\beta}$ and high positive value of $\hat{\gamma}$ both make intuitive sense as described previously. In Figure 1, we show a fit of actual CDS MTM at time $t=1$ v/s estimated CDS MTM at time $t=1$.

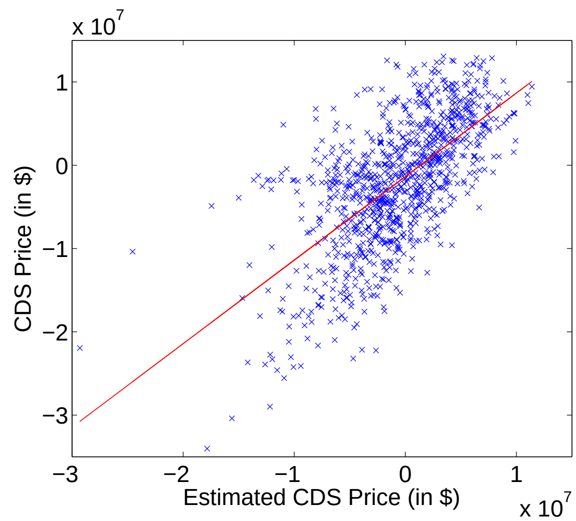

Figure 1. Regression of Actual CDS MTM vs Estimated CDS MTM

We now need to choose appropriate hedging instruments. Since a wide variety of exchange-traded interest rate contracts are available in the market, we run a regression between interest rates and CDS prices at time $t=1$ and notice a clear linear relationship between the two. Hence, we choose CME Eurodollar futures as the interest rate hedging instrument. CME Eurodollar futures contracts are highly liquid standardized exchange traded contracts with almost no counterparty risk and are well-suited to act as a hedging instrument. Since Eurodollar future prices are given by $100 - r$, they have an almost linear exposure to interest rates and hence, are well-suited for hedging interest rate exposure of CDS. The notional amount of Eurodollar futures to enter into is calculated as $N_{ir} = -\frac{D^{ir}_{cds}}{D^{ir}_{fut}} = \hat{\beta}$ so as to make the hedge portfolio duration-neutral.

The regional diversification of mortgages underlying MBS and CDO instruments means that CDS instruments have exposure to US-wide housing market. As such, we choose CME Housing Composite Index futures to hedge house price exposure in CDS contracts. For clarity, we describe contractual details of the instrument briefly below.

The S&P/Cash-Shiller home price indices are designed to be a consistent benchmark of housing prices in the United States. They measure the average change in single-family home prices in a particular geographic market. The indices are calculated with the repeat sales method, which uses data on properties that have sold atleast twice, in order to capture the true appreciated value of constant quality homes. The main variable used for index construction is the price change between two arms-length sales of the same single-family home. The S&P/Cash-Shiller National Home Price index is a composite of single-family home price indices for the nine US Census divisions and is calculated quarterly (S&P Indices, 2011).

The S&P/Cash-Shiller home price index is tradeable through futures on the Chicago Mercantile Exchange, known as Housing Composite Index (HCI) Futures. These futures are cash-settled and the underlying is 250 times the S&P/Cash-Shiller National Home Price index. Typically, contracts expiring during the next 6 quarters are available to trade at any given point of time.

From empirical data, we observe that bid-ask spread for front-end expiring HCI Futures is approximately 4% of notional. Using trading volume data for 2012 and 2013, we observe that approximate annual trading volume is 300 contracts. The bid-ask spread and annual volume most likely make HCI Futures infeasible as a hedging instrument but we hope that greater awareness, more quotes from other traders, and a willingness by traders and hedgers to dabble in these products will improve liquidity of futures on this critically important financial market. (Dolan, 2011)

Since HCI futures duration can be written as $D_{hpi}(t) = \exp((r_t - q) * (T - t))$, where q = dividend yield on HPI and T = settlement time of HCI future, we can calculate the notional amount of HCI futures to enter into as $N_{hpi} = -\frac{D^{hpi}_{cds}}{D_{hpi}} = \exp(-(r_0 - q) * T) * \hat{\gamma}$. We obtain $N_{ir} = -4.29 * 10^8$ and $N_{hpi} = -2.34 * 10^6$.

Finally, hedge portfolio MTM can be calculated as $CDS^H_i = CDS_i + N_{ir} * IR^{MTM}_i + N_{hpi} * HPI^{MTM}_i$, where IR^{MTM}_i= MTM of Eurodollar futures contract along path i at time $t = 1$ and HPI^{MTM}_i= MTM of HCI futures contract along path i at time $t = 1$. We use the Monte-Carlo paths not used to calculate durations to evaluate the performance of our hedge portfolio over the next period. We observe that 1. the standard deviation of MTM portfolio value at time $t = 1$ decreases from $8.02 * 10^6$ to $4.69 * 10^6$, and 2. we eliminate fat tails to quite a large extent. The strategy described above can now be reused to update the hedge portfolio at time $t = 1$. The Figure 2 shows the probability distribution of the hedged and unhedged portfolio.

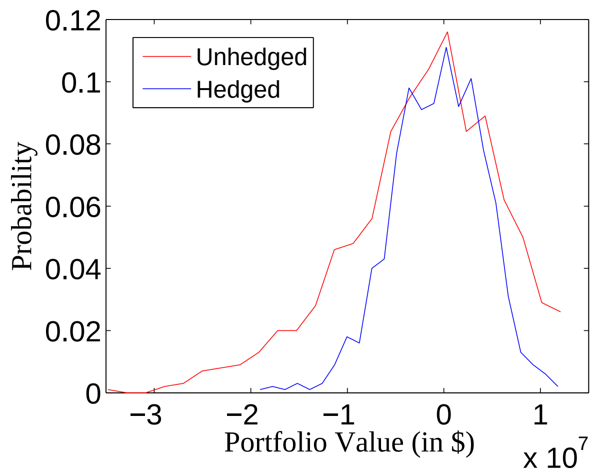

Figure 2. Probability Distribution of Hedged and Unhedged Portfolio Value

4.3 Hedging with Puts on Equity

AIG's leveraged positions accumulated to a large exposure to the mortgage market. The total credit exposure of the CDS position represents a large portion on subprime mortgages trading on the market. On the other hand, the supply chain of the subprime mortgage market in the US can be divided to just a few parts: origination, aggregation and securitization. The subprime residential mortgages are created through two major sources: wholesale and retail. US subprime mortgage market is highly concentrated in the sense that more than 60% of the origination are dominated by a few large wholesale originators: banks, thrifts and unaffiliated mortgage originators. (Stanton et al., 2014)

According to a report of The Home Mortgage Disclosure Act (HMDA), the top forty lenders account for more than 90% of the residential mortgage origination in 2006. The top 10 lenders accounted for more than 60% origination in 2006 . These mortgage originators were highly leveraged on short term financing using Repo or Asset Backed Commercial Paper market. Because asset backed securities account for a large portion of their balance sheet before securitization. Therefore essentially they have similar credit exposures as the CDS AIG had underwritten. For some independent mortgage companies, such as Countrywide Financial Corp and New Century Financial Corp, the entire business was based on the subprime mortgage origination. A natural hedging bet would be that when subprime mortgage market suffers, the stock prices of these companies will also drop. We want to buy a basket of 10% out of the money put options to hedge AIG's large CDS exposures on subprime mortgage CDO. Considering the size of the CDS exposure that AIG had, there were very limited instruments that can hedge against credit risk of subprime mortgages.

We collect the following data for the study of this empirical relationship.We downloaded the ABX AAA 2005-2 weekly index from Bloomberg during period between 2007/8/31 to 2013/9/27. We obtain the public traded put option data for companies in HMDA's top 40 originators. They are: Countrywide Financial Corp. , Wells Fargo Co., Washington Mutal Bank, Citigroup, JPMortgage Chase Corp., Bank of American Corporation, Wachovia Corp., GMAC Residential Capital Group, Indymac Bank, GMAC Residential Holding Corp, EMC, SunTrust Bank, PHH group, Capital One Group, BB&T, New Century Financial, National City Corp, US Bancorp and Mortgage IT. The option data are obtained from OptionMetrics through WRDS. For each company, we select all the put options to be at least 10% out of the money and open interest larger than 600.

The empirical study in Figure 3 shows a clear negative correlation between the ABX prices and put option prices for the full sample period from 2007. Since we do not have the real market data on the CDS position AIG held, this study at least will tell us how robust the hedge is. for subprime mortgage market in general We notice that our data may be biased after 2008 financial crisis as a lot of companies have shrunk their holdings of the asset backed securities. For example, the private label MBS market is still very light. But in general, we see ABX as an import global economic indicator for the US housing market and should be positively correlated with the financial sectors. Based on the regression result, put option can achieve hedging to cover 1.5 times of ABX index loss on average. The adjusted Rsquareds distribute from 1% to 16%. If we choose a subset of the companies that had more than 10% Rsquared and perform a multi variate regression of ABX returns on put options, we essentially create a minimized variance hedging portfolio. Our empirical study shows the residual variance of the hedged portfolio is less than 40% of the unhedged portfolio. Although there are not enough DOOM put options on the market to cover the entire huge CDS exposures, we still believe this hedging method should at least cover some exposure with reasonable prices of the cost.

5. CVA-based Collateral Agreement

5.1 Collateral, The Economics Behind

Let's consider a simple contract with counterparty risks. We denote the two counter parties A and B. The contract has a lump sum at time T. The lump sum payoff is stochastic, with a symmetric distribution with zero mean, for example the payoff can be a drift-less Wiener process at T. The expected payoffs for both parties are 0 at T without considering the counterparty risks.

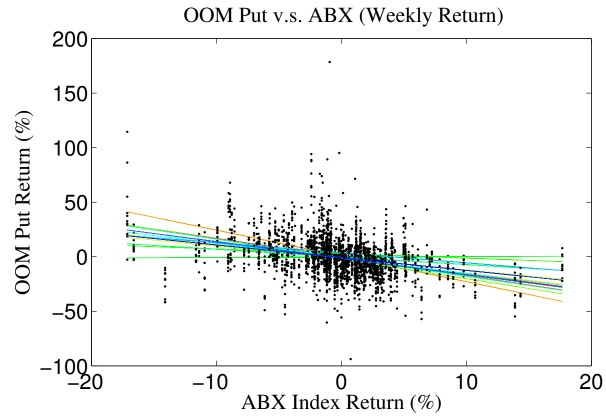

Figure 3. ABX Put Return

Now let's consider the scenario that A is default-risk free, but B has a default risk. Now, A faces the counterparty risk from B, with a non-zero probability that B defaults when the contract matures in money for A at time T. It follows that the value of the contract to A is less than 0 at $t = 0$. The present value of A's expected loss for this contract, which is the definition of the credit value adjustment is

$$\mathbf{CVA}_A = DF(0,T)\cdot\mathbf{LGD}\cdot PD_B^Q(0,T)\mathbb{E}^Q\left[\max(V_T,0)\right] \quad (12)$$

where $\mathbf{CVA}_A$ is the credit value adjustment to party A, **LGD** stands for the loss given default, $PD(0,t)$ is the probability of default and V_T is the value of the contact.

At time $t = 0$, the value of this contract in A's point of view is $-\mathbf{CVA}_A$, since the expected payoff is 0 without counterparty risk, and there is no initial cash exchange. However, from B's point of view the contract still has value 0. In other words, the contract is unfair for A.

There are several ways to remedy B's credit risk. The first way is that B pays half of $\mathbf{CVA}_A$ to A at $t = 0$, so that the values of the contract to both parties are the same.

The second way is that B can post full mark-to-market collateral to A. We assume that the collateral is posted continuously, default of B is recognized immediately, and there is no additional cost to the mark-to-market price fro A to replace the contract. In case of B defaults, since A can always replace the original contract with other party using the collateral B posted. In this case, the expected loss for A is 0, and so is $\mathbf{CVA}_A$. The contract value is 0 to both parties again.

Now we consider the case of both A and B having default risk. B's expected loss is similar to A, with reversed sign of payoff.

$$\mathbf{CVA}_B = DF(0,T)\cdot\mathbf{LGD}\cdot PD_A^Q(0,T)\mathbb{E}^Q\left[-\min(-V_T,0)\right] \quad (13)$$

The contract value for A and B are

$$V_A = V_0 - \mathbf{CVA}_A \quad (14)$$
$$V_B = -V_0 - \mathbf{CVA}_B \quad (15)$$

For a fair trade, we have $V_A = V_B$, or

$$2V_0 - \mathbf{CVA}_A + \mathbf{CVA}_B = 0 \quad (16)$$

If A and B has different probability of default, $\mathbf{CVA}_A$ and $\mathbf{CVA}_B$ will not equal. For equation (16) to hold, there are several ways. For example, A and B can arrange initial cashflows, so the V_0 is non-zero. Alternatively, they can adjust their collateral, so that $\mathbf{CVA}_A = \mathbf{CVA}_B$, and V_0 can be kept at 0. By adjusting their collateral, we meant adjusting the collateral scheme. A collateral scheme is a rule decided at the inception of the contact of determining the amount of collateral that each party has to post under different market conditions.

There can be different collateral schemes, for example, a fixed percentage of the mark-to-market value of the contract. A more complex collateral scheme will consider not only the mark-to-market value but also the counterparty risk value adjustments. However, the value of the contract becomes hard to estimate under this scheme, because the counterparty risk value adjustment depends on the collateral, which in turn depends on the counterparty risk value adjustment. In this study, we only consider posting fixed percentages of mark-to-market value as collaterals without considering the counterparty risk value adjustment.

Now we return to our simple contract, and invest how much collateral each party need to post. Suppose A has decided to post a fraction of α_A of mark-to-market value as collateral, the question is what fraction α_B of mark-to-market value B should post. We consider the simplest case, in which the default hazard rates of A and B is constant during the contract time.

From $\mathbf{CVA}_A = \mathbf{CVA}_B$, we can rearranging

$$(1-\alpha_A)\left(1-e^{\lambda_A T}\right) = (1-\alpha_B)\left(1-e^{\lambda_B T}\right) \quad (17)$$

We calculated the relationship of α_A and α_B, with different combinations of counterparty risks. Figure 4 shows the relationship of collateral percentages when an "A" rating party are facing counterparty with different ratings.

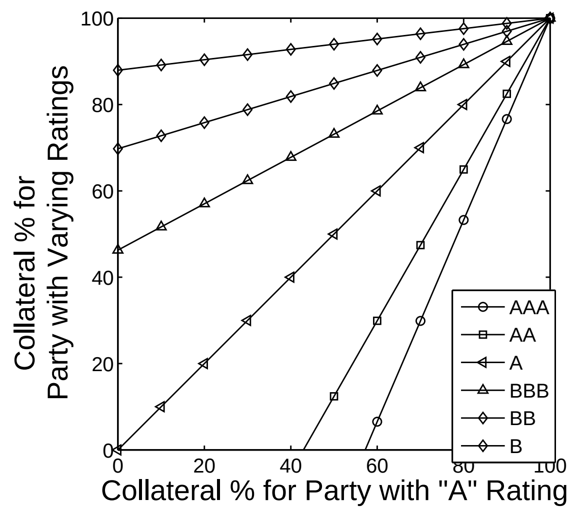

Figure 4. The collateral percentage relationship of an A rating party facing counterparty with different ratings

5.2 Implementation of CVA

Discretizing the CVA expression in Equation 12, we get

$$\mathbf{CVA}(0) = (1-R)\sum_{i=0}^{T-1}\mathbb{E}^{Q}\left[\frac{1}{2}\left(E(t_i)DF(0,t_i)\right.\right. \quad (18)$$
$$\left.\left. + E(t_{i+1})DF(0,t_{i+1})\right)\left(S(t_i)-S(t_{i+1})\right)\right] \quad (19)$$

From the above relationships, it is evident that we need the value of the CDS contract at future times to calculate CVA.

The most obvious but computationally expensive methodology (and hence not possible in practice) to calculate CVA is simulating multiple paths from each future node of the original simulated paths (originating at $t = 0$) and using equation 21 to get the CVA at each node. In order to avoid simulations inside simulations we propose application of the Least Square Monte Carlo (LSM) approach proposed by Longstaff and Schwartz (2001) to calculate CVA at future times.

The LSM approach proposes that the expected continuation value of an American contract at time t and node i of the simulated paths can be estimated by doing least square regression on the pathwise discounted values of the realized cash flow. The fitted values from this regression are the expected continuation values. This approach is generic and can be used to calculate the expected future values of any contract. Therefore, in our study, we use LSM approach to calculate the expected future value V_t of the CDS contract.

Choosing the right regressors is the key to improving the accuracy of the fitted values in the LSM approach. We note that the value of our CDS contract fundamentally depends on the House Price Index (HPI) and interest rate (r). We, therefore, do the following regression at each time t, to get $V_t ateachnodei$:

$$\mathbf{DCF}(t) = \alpha + \beta_1 H_t + \beta_2 H_t^2 + \beta_3 H_t^3 + \quad (20)$$
$$\gamma_1 r_t + \gamma_2 r_t^2 + \gamma_3 r_t^3 + \quad (21)$$
$$\theta_1 \log H_t + \theta_2 (\log H_t)^2 + \theta_3 H_t r_t \quad (22)$$

where **DCF** is the discounted sum of the realized cash flows from time t to T (maturity of the contract) along the given path. Its worth mentioning that adding the log terms in the above regression significantly improved the regression quality in comparison to that of the regression done without log terms.

In addition to V_t at each node n of the simulated paths, we also need the risk neutral probabilities of the counterparty default. In practice, these can be calculated from the CDS on the counterparty. However, for simplicity, we have used constant hazard rates for default probabilities calculation. Denoting probability of default between t and Δt by $PD(t, t+\Delta t)$ and survival probability uptil time t by $SP(t)$, we note that,

$$PD(t, t+\Delta t) = SP(t) - SP(t+\Delta t) \tag{23}$$

where we assume a constant hazard rate over the life the firm, hence

$$SP(t) = \exp(-\lambda t) \tag{24}$$

We calculate PD between different intervals using the constant hazard rates obtained from the 4.5 years implied PD for different rating in the paper by Terry and Andrew.

Using the above methodology, the CVA for the CDS buyer is \$77485.61 and that for the CDS seller is \$19724.19 assuming that both have AAA rating and neither party post collateral.

As described in the previous section, we calculate the collateral for the CDS contract at time t as x percentage of the value at time t where x is calculated such that CVA for the seller equals the CVA for the buyer. In our example, as expected, the CVA for the buyer is greater than the CVA for the seller. Therefore, the seller of the CDS contract will post collateral such that the CVA of the buyer reduces and becomes equal to the CVA of the seller.

The graph depicts that as the credit rating of the CDS seller depreciates, it is required to post greater percentage of the contract value as collateral. The buyer rating is assumed to be AAA for this example.

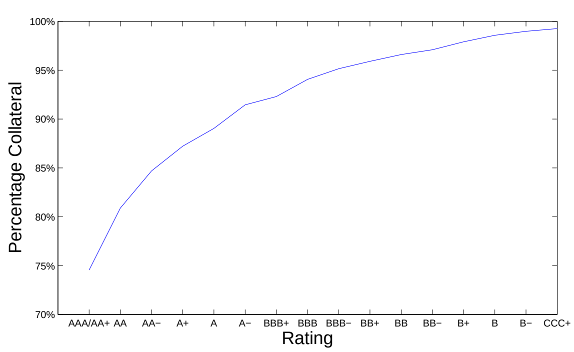

Figure 5. Collateral Percentage Variation with Seller Rating

6. Capital Reserve of CDS

A firm faces two kinds of losses on the CDS contract: i) the mark to market losses on the contract value and ii) mark to market of the CVA Denoting time by t, value of the contract at t by V_t, collateral posted by counterparty at t by C_t, we can write the Profit and Loss equations:

$$PnL_1 = V_{t+1} - V_t - C_t$$

$$PnL_2 = CVA_t - CVA_{t+1}$$

Note that CVA is a loss by definition and hence $PnL_2$is defined by subtracting the latest CVA value from the previous value. For the purpose of illustration, we do the capital reserve calculation only for the seller of the CDS contract for a horizon of a quarter. Note that the two losses described above are correlated as both depend upon contract value. We already have 2000 samples of V_{t+1} (using the LSM approach described in the above section). For each V_{t+1}^i where i represents the ith path we calculate the total loss $PnL^i = PnL_1^i + PnL_2^i$ and get the total loss distribution at t+1. In order to calculate PnL_2^iwe need to get the CVA values at t+1. We once again use LSM approach to calculate CVA at each node n at $t = 3$ months. CVA at $t = 3$ months is the predicted value of the regression below:

$$\mathbf{CVA}(t) = \alpha + \beta_1 H_t + \beta_2 H_t^2 + \beta_3 H_t^3 + \tag{25}$$

$$\gamma_1 r_t + \gamma_2 r_t^2 + \gamma_3 r_t^3 + \tag{26}$$

$$\theta_1 \log H_t + \theta_2 (\log H_t)^2 + \theta_3 H_t r_t \tag{27}$$

For estimation of the regression coefficients we use the path-wise CVA values based upon the realized cash flows along a particular path.

Capital Reserve is 1% percentile (i.e. 99% VaR) of the total PnL distribution.The graphs below illustrates that with our collateral scheme the variation of the CVA loss distribution and the hence the CVA capital reserve requirement are stable for various possible ratings of the CDS seller. This result is expected as our collateral scheme requires low rated CDS sellers to post more collateral than is posted by high rated CDS seller. Eventually the CDS buyer face the same CVA irrespective of the rating of the CDS seller. However if we consider a fixed collateral posting (Figure 6), the variation in the CVA losses increases as the rating of the CDS seller depreciates. Hence the capital reserve requirements for the CDS buyer will increase.

It worth mentioning that the PnL distribution in the Figure 10 looks same. However, they are similar but not the same. The graph below shows the 1% VaR of the PnL distribution and clarifies the presence of some variation amongst the PnL distribution. In the Figure 6, The red vertical line represents 99% VaR.

We illustrated the concept of capital reserve only for one product. However, in practice, we need to take into account the net PnL of the firm taking into consideration all the positions the firm is holding and carefully netting while modeling correlation between various positions.

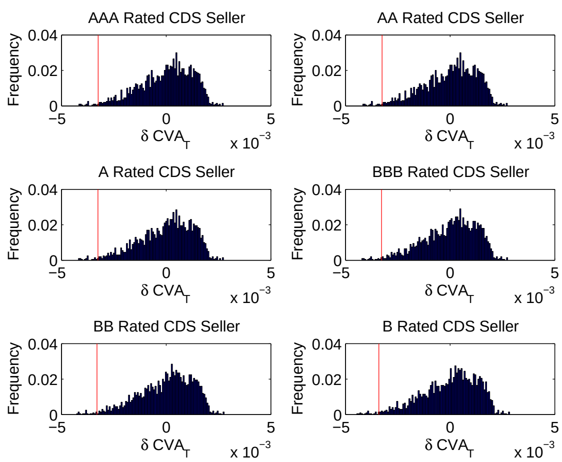

Figure 6. Distribution of CVA PnL for Different Seller Ratings with the Proposed Collateral Scheme

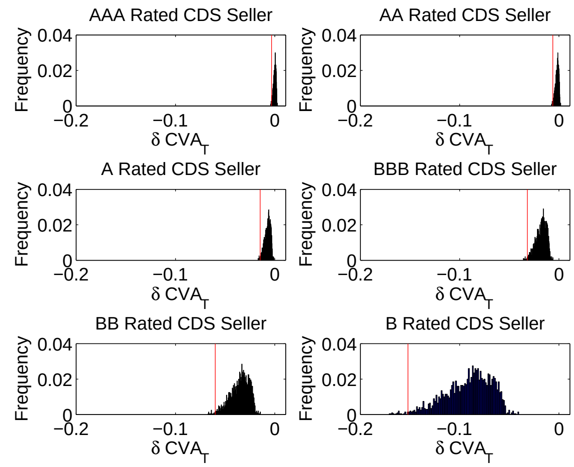

Figure 7. Distribution of CVA PnL for Different Seller Ratings with the 75% Collateral Scheme

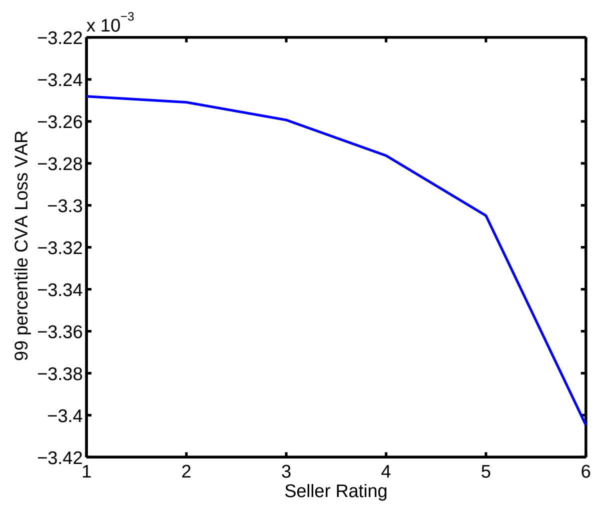

Figure 8. 99% VaR of CVA with Different Sellers Rating

7. Conclusion

In this study, we revisited the series of events that leads to the bailout of AIG. The AIG's near-collapse is rooted with its huge cumulated position in the CDS contracts on the super senior tranches of CDOs, which were primarily backed by subprime MBS. We implemented a macro-economical factor model to simulate the cash flows of such CDS contract based on HPI and interests rate. Schemes were proposed to hedge CDS contact on MBS by ABX.HE indices, put options on mortgage market participants, and vanilla Eurodollar and HPI futures. Simulations and hedging results are presented. We carried on to model the counterparty risk of this type of CDS contract and the effects of collateral schemes on the CVA valuation. We demonstrated a framework to calculate the VaR on the CVA, and proposed a model of using the CVA VaR as capital requirements for counterparty risks.

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Appendix

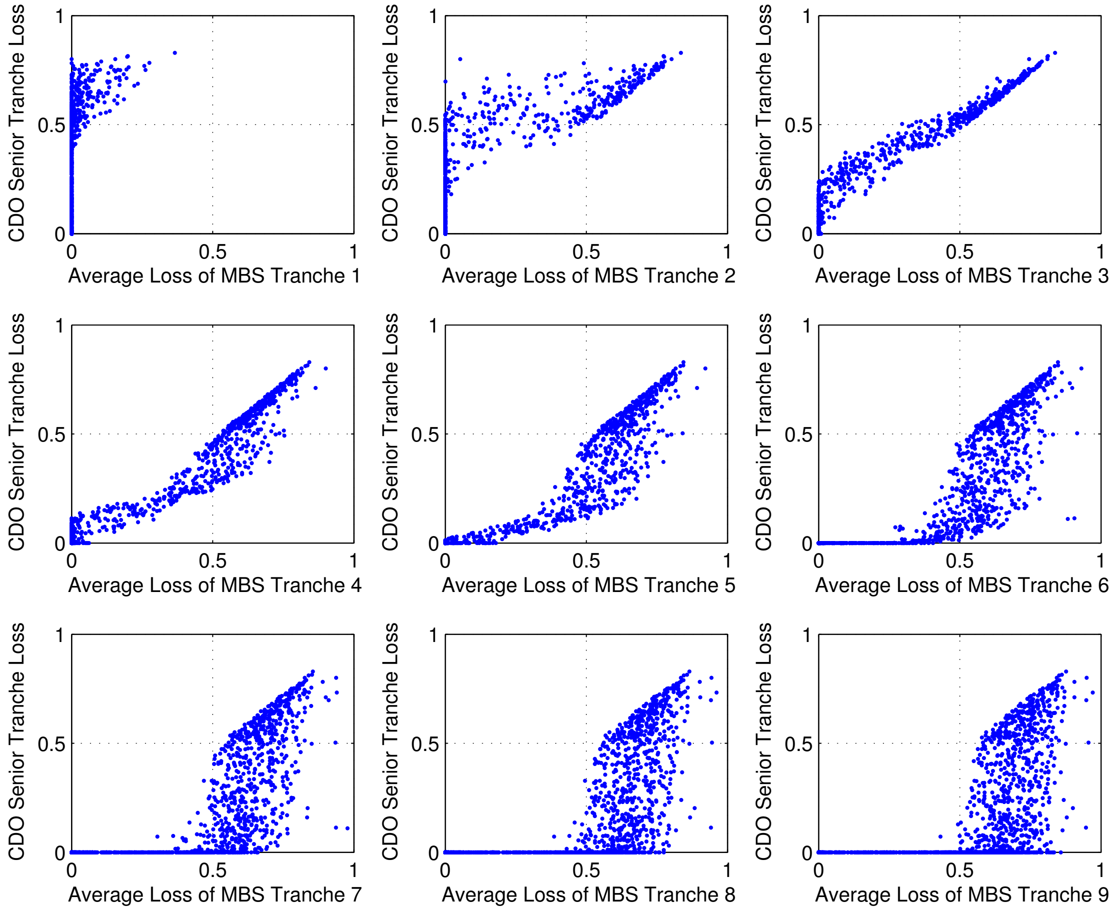

Figure 9. The joint distribution of the loss on the CDO senior tranche and average loss of different MBS tranches

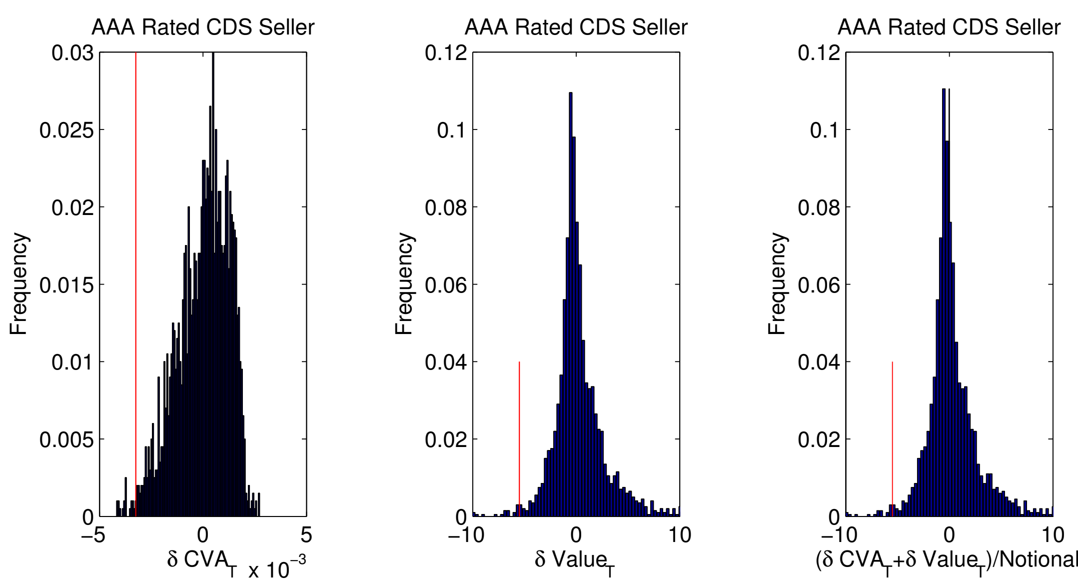

Figure 10. Decomposition of Net PnL into PnL_1 and PnL_2

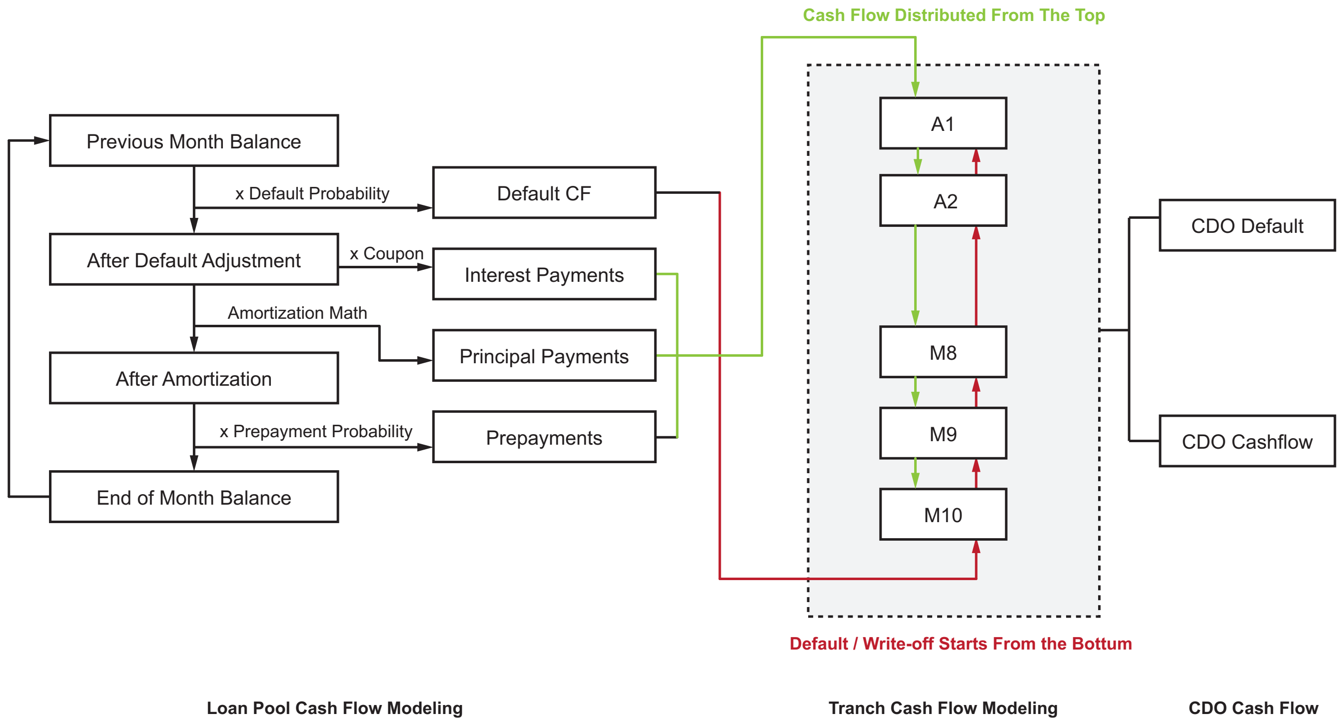

Figure 11. Schematic Design of MBS Cash Flow Modeling

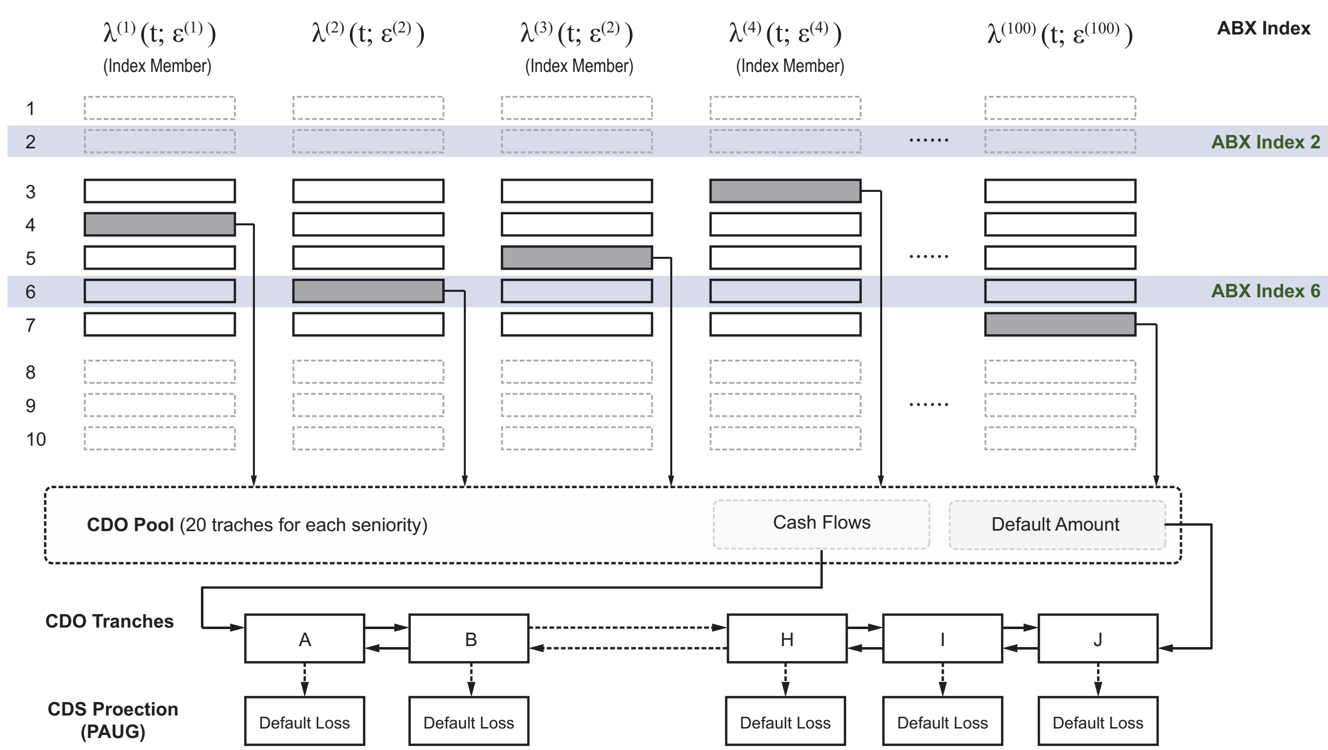

Figure 12. Schematic Design of CDOs, ABX Cash Flows Modeling