Forecasting Credit Spreads: A Machine Learning Approach

Abstract: In this paper, we managed to develop a LSTM model with 88.9% out-of-sample accuracy in predicting the direction of next day credit spread change. We constructed and compared the outputs of several different models with verified features provided by previous literature. Starting with Vasicek model, then tree models including Random Forest as well as BART (Bayesian Additive Regression Tree) and finally LSTM (Long Short-Term Memory) model, the model performance and prediction power was improved step by step. BART model worked well in predicting the magnitude of credit spread but fail to predict the direction accurately. However, LSTM model with 3-day look-back period along with feature variables yielded the best prediction of future credit spread change direction. Besides, we captured two main characteristics of credit spread prediction. First, credit spread can be well explained by its own history. Second, model should be able to evolve and gradually forget older data, while also learning patterns from more recent history in order to capture regime changes. Specific machine learning details and economic interpretations of features are also discussed.

Keywords: credit spread, prediction, LSTM, BART, Random Forest

Contents

1 Introduction
   1.1 Economic Significance . . . . . . . . . . . . 1
   1.2 Literature Review . . . . . . . . . . . . . . 1
   1.3 Model Evolution . . . . . . . . . . . . . . . 2

2 Data

3 Methodology and Model Evaluation
   3.1 Vasicek Model . . . . . . . . . . . . . . . . 3
   3.2 Random Forest . . . . . . . . . . . . . . . . 4
   3.3 BART Model . . . . . . . . . . . . . . . . . 5
   3.4 LSTM Model . . . . . . . . . . . . . . . . . 6

4 Conclusion
   4.1 Results . . . . . . . . . . . . . . . . . . . . 8
   4.2 Potential Improvement . . . . . . . . . . . . 9

1. Introduction

1.1 Economic Significance
Credit spreads represent the difference in the yield of a risky security and a risk-free security of similar maturity. It acts as a good indicator for both the credit rating of a firm and also the condition of the financial market. For example, in Figure 1, the credit spread is computed as the difference between US corporate AAA bond yield and 10-year treasury constant maturity rate. The result is plotted in Figure 2. Figure 2 shows that the credit spread is decreasing from 2001-2002, which implies that the market is transitioning from recession to economic expansion. After the Dot-com bubble in 2000, the U.S. government lowered the interest rate in order to stimulate the economy. The second grey area in Figure 2 corresponding to the 2008 great recession, shows the credit spread spiking upward, indicating an economic downturn.

Thus, predicting the direction and magnitude for credit spreads effectively makes an important role in financial market. Credit spread can serve as a useful tool for fixed income investors and corporate finance professionals. Fixed income investors can make a profit from abnormal trading by predicting credit spreads effectively. For example, if an investor forecasts that the credit spread will widen in the future, then they could sell the corporate bonds and buy government bonds. Alternatively, if they forecast that the spread...
will tighten in the future, then they can sell government bonds and buy corporate bonds. In the end, the investor will sell bonds at a higher price but buy with a lower price. And the difference is the profit that the investor earns.

Corporate finance professionals can reduce the cost capital. For example, if a firm is planning to issue long-term corporate bonds and if the firm predicts that the credit spread will tighten in the future, to avoid the high cost of interest rate, the firm could fund with short-term debt and wait until credit spreads have actually narrowed.

1.2 Literature Review

Credit spread is widely used as a symbol for systematic risk, liquidity, and conditions in financial economics. We find in previous papers that the direction-of-change and magnitude of the credit spread are explained well by bond characteristics, credit quality, and market conditions. The conclusions of the structural approach regarding the shape of the term structure of credit spreads are confirmed by the empirical work by Huang[1]. Option-implied volatility is also a measure for future cash flow uncertainty as Chiras and Manaster (1978)[2] mentioned. Pierre Collin-dufresne, Robert S. Goldstein, and J. Spencer Martin (2001) summarized that changes in leverage had significant explanatory power for the movement of credit spread[3]. The market jump risk factor constructed from high-frequency data also seems to capture low-frequency movements in credit spreads in terms of long-run trends, as shown by George Tauchen, Hao Zhou (2011)[4]. To capture changes in the overall economy, they used quarterly S&P 500 index.

Hai Lin, Sheen Liu, and Chunchi Wu developed a new methodology to better dissect the components of credit spreads and described their findings. They discover that 47% of yield spreads are explained by default risk while the remaining 53% is made up of nondefault risk[5]. Different patterns exist on different time-regimes. Over the short-run, credit spreads are negatively related to treasury rates. Initially, spreads narrow because a given rise in treasuries produces a proportionately smaller rise in corporate rates. However, over the long-run, this relation is reversed. A rise in treasury rates eventually produces a proportionately larger rise in corporate rates. This widens the credit spread and induces a positive relation between spreads and treasury rates[6].

1.3 Model Evolution

Many people believe financial markets are unpredictable. However, there are still many methods proved to be successful on capturing dynamics of financial markets. Mathematicians use stochastic models to explain the dynamics of financial markets. Geometric Brownian Motion (GBM) is one of the earliest and simplest model used to describe stock prices, which is still widely today by financial firms. CIR and Vasicek models are used to describe dynamics of the interest rate and the Heston model is used to describe volatility of underlying asset. Since credit spreads are based on interest rate of government and corporate bonds, it is natural to use mathematical equations like Vasicek model to fit the dynamics of credit spread.

Although a well defined stochastic differential equation is a good starting point, typically it cannot completely capture the whole dynamics. One of the reasons is the limitation of parameters. We turn to machine learning, another popular technique used in financial markets. Financial markets were some of the earliest adopters of machine learning and it is still one of the mainstream tools on markets analysis. It shows great success on many market related processes, which leads us to seek suitable machine learning methods on predicting credit spreads. We will focus on some models that have been proved to perform well in financial markets. Our goal is to use these models for predicting the credit spread.

Decision tree is one of the widely used machine learning methods in analyzing financial markets. A decision tree is a decision support tool that uses a tree-like model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility. In this paper, random forest model will be used first in order to get an easy interpretable result. Later, with supporting evidence from Hugh A. Chipman, Edward I. George, and Robert E. McCulloch (2008)[7], we will use Bayesian additive regression tree (BART) model, because it appears to be robust to small changes in the prior distribution and the choice for number of trees. Therefore, BART is introduced to improve the tree model performance.

Finally, Long Short-Term Memory (LSTM) is also used as a comparison. It was shown by Pankaj Malhotra, Lovekesh Vig, Gautam Shroff, and Puneet Agarwal (2015) that by modeling the normal behavior of a time-series via stacked LSTM networks, we can accurately detect deviations from normal, also suggesting that LSTM-based prediction models may be more robust compared to RNN-based models[8].

The above methodologies lay the foundation for our analysis on credit spread prediction. In addition, some potential shortcomings of each approach are also covered.

2. Data

In order to have a good understanding of historical movements of credit spread, the first task lies in determining which historical period we should choose. To capture previous economic expansions as well as recessions, we set our time horizon from December 1996 to January 2019.

The following variables are used to conduct our empirical experiment. Most of them have major capability to predict the movement of credit spread based on previous literature.

- **CS**: Credit Spread of US investment grade corporate bonds (e.g. BofAML US Corporate AAA) over US Treasuries (e.g. 10-Year Treasury Constant Maturity Rate). Data is obtained from the St. Louis Fed data repository and daily observations are used for the period December 1996 through January 2019.
• **SLOPE**: Slope of terms, which is the yield on 10-year Treasury bonds minus yield on 2-year Treasury bonds, which is calculated by taking the difference between 10-Year Treasury Constant Maturity Rate and 2-Year Treasury Constant Maturity Rate. Daily observations are obtained from the St. Louis Fed data repository.

• **VIX**: Cboe volatility index on the S&P 500 index. Daily data is retrieved from Bloomberg. VIX is a barometer of equity market volatility. It is based on real-time prices of options on the SP 500 Index (SPX) and is designed to reflect investors’ consensus view of future (30-day) expected stock market volatility. The VIX Index is often referred to as the market’s “fear gauge”. It can be correlated to the credit spread, which which basically captures the future probability of default as a a common forward-looking risk metric.

• **SP500_R**: Return on the S&P 500 stock index. It was calculated by S&P500 index from Yahoo finance by dividing the difference of current day’s S&P 500 stock index and previous day’s S&P 500 index by previous day’s S&P 500 index. The equity market could be relevant to corporate bond credit spreads in several ways: as an alternate investment, as a measure of capital markets investment levels, or as a yield-producing security. The equity market as an alternative to fixed income investment is the most plausible choice in today’s environment.

• **SWAP_SPREAD**: A swap spread is the difference between the fixed rate component of a given swap and the yield on a Treasury item or other fixed-income investment with a similar maturity, here a corresponding 10-year maturity is chosen and daily data is retrieved from Bloomberg. Swap spread is highly correlated with credit spread because the swap rate is a proxy for credit rate. As the swap market is significantly deeper and more liquid than for corporate bonds, swap rates may provide a forward indication of the direction of credit spreads. As swap spreads increase, so should credit spreads[9].

• **SKEW**: The Cboe SKEW Index (“SKEW”) is an index derived from the price of SP 500 tail risk. The price of SP 500 tail risk is calculated from the prices of SP 500 out-of-the-money options. The SKEW Index can reflect the tail risk in the equity market.

The following data are generated in order to achieve two purposes. First, as shown in Figure 3, the collinearity between the variables above will result in poor performance of parameter estimation, for example the correlation between VIX and spread is more than 0.5; therefore, we first-differences of the original data to make the time-series more stationary and less collinear. Second, historical credit spread may act as a good feature in prediction. Here, we provide the partial autocorrelation function (PACF) plot. PACF gives the partial correlation of a time series with its own lagged values, controlling for the values of the time-series at all shorter lag orders. As shown in figure 4, the partial autocorrelation of lag-1, lag-2 and lag-3 are significant. Hence lag-i values of spread are also listed.

3. Methodology and Model Evaluation

3.1 Vasicek Model

The Vasicek model is a mathematical model describing the evolution of interest rates. It is a type of one-factor short-rate model that describes interest rate movements driven by only one source of market risk. The model can be used in the valuation of interest rate derivatives and has also been adapted for credit markets. It was introduced in 1977 by Oldřich Vašíček and can also be seen as a stochastic investment model. The model specifies that the instantaneous interest rate follows the stochastic differential equation:

\[ dr_t = a(b - r_t)dt + \sigma dW_t \]  

where:

• \( b \): "long term mean level". All future trajectories of \( r \) will evolve around a mean level \( b \) in the long run.
• \( a \): "speed of reversion", which characterizes the velocity at which such trajectories will regroup around \( b \) in time.
• $\sigma$: "instantaneous volatility", which measures instant by instant the amplitude of randomness entering the system. Higher $\sigma$ implies more randomness.

The model calibration is conducted as follows: First, the continuous SDE could be discretized as follows:

$$CS_{t+\Delta t} - CS_t = a(b - CS_t)dt + \sigma \sqrt{\Delta t} \epsilon_t$$  \hspace{1cm} (2)

provided that $\epsilon_t$ is i.i.d. $N(0, 1)$. Then a simple linear regression is used to yield an estimation of the parameters above. To be more specific, the model is calibrated every 100 days through a rolling window; in this way, the parameters are allowed to change over time. Figure 1 shows the movement of $\sigma$ and $b$.

Based on Figure 5, we can see that parameter values change suddenly at certain time periods, especially around year 2008, and then go back to normal quickly. This indicates that the Vasicek model is not time-homogeneous for credit spread, in other words, one single set of parameters does not hold for all time regimes. Using only one set of parameters could be detrimental to our predictions, because the estimated parameters would not adapt to regime changes in future times. Therefore, in order to capture this property, we need more complex models.

### 3.2 Random Forest

Vasicek model only takes the historical data of credit spread into account. In order to predict the future direction of credit spread, we use a classification method and add more variables mentioned in our literature review such as S&P 500 return, VIX, and so on. The decision tree is fundamental to our random forest.

Classification and regression trees or CART models, also called decision trees, are defined by recursively partitioning the input space, and defining a local model in each resulting region of input space. This can be represented by a tree, with one leaf per region.

The model can be written in the following form:

$$f(x) = \mathbb{E}[y|x] = \sum_{m=1}^{M} w_m 1(x \in R_m) \hspace{1cm} (3)$$

Here, vector $x$ represents our features, the label as $y$, $R_m$ is the $m$-th region, $w_m$ is the mean response in this region, and $v_m$ encodes the choice of variables to split on.

We can associate a mean response with each of these regions, resulting in the piecewise constant surface shown in Figure 6. The technique known as random forests (Breiman 2001a) tries to decorrelate the weak learners by learning trees based on a randomly chosen subset of input variables, as well as a randomly chosen subset of data cases.

Random forest is a bagging method. Bagging (stands for Bootstrap Aggregating) is a way to decrease the variance of prediction by generating additional data for training from the original data set using combinations with repetitions to produce multi-sets of the same cardinality size as the original data. We cannot improve the model’s predictive accuracy by increasing the size of the training set, but only decrease the variance, narrowly tuning the prediction to the expected outcome.

Bagging is a frequentist concept, but it is possible to adopt a Bayesian approach to learning trees. In particular, (Chipman et al. 1998; Denison et al. 1998; Wu et al. 2007) perform approximate inference over the space of trees (structure and parameters) using MCMC. This reduces the variance of the predictions.

In order to predict the next-day direction-of-change for the random forest model, we use the following 9 variables: SLOPE, VIX_DIFF1, SP500_R, SWAPSPREAD,
we lose some important information of the continuous label. Later on, we will use the boosting method instead of bagging and turn to the regression model rather than the classification model.

### 3.3 BART Model

Since the random forest model has poor performance on prediction, we want to improve our tree model with better ensemble methods. Thus, we introduce a boosting tree model called: Bayesian Additive Regression Trees (BART) Model. In contrast with the random forest model, BART uses a boosting method to train its model. BART is a stronger learner for each tree in the model. Additionally, boosting is a great method for dealing with underfitting.

In the BART model, each tree is constrained by a regularization of the prior to be a weak learner. Fitting and inference are accomplished via an iterative Bayesian back-fitting MCMC algorithm that generates samples from a posterior. In the following we will briefly introduce the BART model.

First, we define inputs as \( x = (x_1, \ldots, x_p) \) which is a \( p \)-dimensional vector. Then, an unknown function \( f \) used to predict output \( Y \). We have:

\[
Y = f(x) + \epsilon, \quad \epsilon \sim N(0, \sigma^2)
\]  

We consider \( f \) as a conditional expectation \( f(x) = E(Y|x) \), and it is approximated by a sum of \( m \) regression trees \( f(x) \approx h(x) = \sum_{j=1}^{m} g_j(x) \) where each \( g_j \) denotes a regression tree. Thus, we approximate the equation above by a sum-of-trees model:

\[
Y = h(x) + \epsilon, \quad \epsilon \sim N(0, \sigma^2)
\]  

The essential idea for this model is to modify the sum-of-trees model by imposing a prior that regularizes the fit by keeping the individual tree effects small. Even though every tree explains a small and different part of \( f \), the sum produces a strong explanation for the output.

We use \( T \) to denote a binary tree that contains a set of interior node decision rules and a set of terminal nodes. Also, \( M = \{\mu_1, \mu_2, \ldots, \mu_b\} \) denotes a set of parameter values in each of the \( b \) terminal nodes of \( T \). With the sequence of decision rules from top to bottom, each input \( x \) is assigned to the \( \mu_i \) value associated with this terminal node. Thus,

\[
Y = g(x; T, M) + \epsilon, \quad \epsilon \sim N(0, \sigma^2)
\]  

is a single tree model.

With the notation from above, the BART model can be expressed as

\[
Y = \sum_{j=1}^{m} g(x; T_j, M_j) + \epsilon, \quad \epsilon \sim N(0, \sigma^2)
\]
where \( m \) denotes the total number of trees.

As for specification of the parameters of the sum-of-trees model, namely \((T_1, M_1), \ldots, (T_m, M_m)\) and \( \sigma \), the model imposes a prior over all the parameters. The model also has independence and symmetry properties. We omit the details for specification of the parameters to save space.

Now we use BART to make predictions on the credit spread. The input \( x \) has the 9 features we mentioned before. The credit spread data is divided into two parts: training set and test set, and the proportion is 7:3. One of BART’s advantages is that it can be used for model-free variable selection. Thus, we do not need a validation set. We get predictions for both training set and test set. We compare predictions for each model in Figure 16.

From the confusion matrix in Figure 11, we see that BART predicts true negatives better than true positives. However, both predictions are around 50% correction. Again this prediction accuracy is akin to a coin toss. Thus, we need a better model that can predict both magnitude and direction-of-change of the credit spread.

### 3.4 LSTM Model

In order to capture the direction-of-change for the credit spread, we continue to test other models. In this case, Neural Network is a good choice. In the conventional feed-forward neural networks, all test cases are considered to be independent. That is, when fitting the model for a particular day, there is no consideration for the data on previous days. In order to capture the time-dependency, we can use Recurrent Neural Networks. However, RNN only remembers for small durations of time. Therefore, we can use Long short-term memory (LSTM), which proves to be an effective solution to this shortcoming.

A recurrent neural network is simply a type of densely connect neural network as shown in figure 12.

![Fig. 12. Structure of a Basic Recurrent Neural Network(RNN)](image)

However, simply using one activation function in each layer can cause certain problem, for example the vanishing
or exploding gradient problem (reference here), and the prediction can be hard for time series data. Therefore, LSTM model was introduced; the key different to normal feed forward networks is the introduction of time - in particular, the output of the hidden layer in a recurrent neural network is fed back into itself.

The structure of a typical LSTM cell is shown in figure 13. Below we illustrate this structure:

![Structure of a Typical LSTM Cell](image)

- "tanh" denotes the hyperbolic tangent function:
  \[ \text{tanh}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]  
  which is used squash the input \( x \) to a value range from -1 to 1.
- \( \sigma \) denotes the sigmoid function:
  \[ \sigma(x) = \frac{e^x}{1 + e^x} \]  
- \( x_t \) and \( h_t \) denote the input and output at time \( t \) respectively.
- \( \times \) and \( + \) denotes the element-wise multiply and add respectively.

The basic concept is as follows: By concatenating \( x_t \) and \( h_{t-1} \), passing through input, forget and output gates respectively, the time dependence and effects of previous inputs are well controlled (remembered or forgotten) and this leads to \( h_t \), which is the output at time \( t \). Below describes the data flow within the LSTM cell:

- **Input**
  At first, the input is squashed between -1 and 1 using \( \text{tanh} \) activation function as follows
  \[ g = \text{tanh}(b_g + x_t U_g + h_{t-1} V_g) \]  
  where \( U_g \) and \( V_g \) are the weights for the input and previous cell output respectively, and \( b_g \) is the input bias.

- **Input Gate**
  The input gate is a hidden layer of sigmoid-activated nodes, which is denoted as a \( \sigma \) in figure 13. Mathematically the expression can be written as
  \[ i = \sigma(b_{input} + x_t U_{input} + h_{t-1} V_{input}) \]  
  After input and input gate, the results are multiplied element-wise as
  \[ g \circ i \]  
  to be simple, the output \( i \) from input gate acts as the weights for input \( g \).

- **Internal State \( S \)**
  This state is delayed by 1 and is added into \( g \circ i \) to provide an internal recurrence loop to learn the relationship between inputs separated by time.

- **Forget Gate**
  The forget gate is another layer of sigmoid-activated set of nodes and is multiplied element-wise by \( S_{t-1} \). As the name suggests, an output close to 1 means to remember and an output close to 0 means to "forget". The result \( f \) can be expressed as:
  \[ f = \sigma(b_{forget} + x_t U_{forget} + h_{t-1} V_{forget}) \]  
  above follows by
  \[ s_t = s_{t-1} \circ f + g \circ i \]  

- **Output Gate**
  The output gate is the final layer of sigmoid-activated set of nodes.
  \[ o = \sigma(b_{output} + x_t U_{output} + h_{t-1} V_{output}) \]  
  So the final output can be expressed as:
  \[ h_t = \text{tanh}(s_t) \circ o \]

For our analysis, multiple LSTM models with different input variables are tested. For the first model, we use the same features as in BART model and also a three-day look-back period. In other words, the credit spread at time \( t-1, t-2, \) and \( t-3 \) are used to feed the model in order to get the prediction of \( \text{CS}_t \). The new model makes a relatively good prediction shown in Figure 14, however it seems to be worse than the BART model.

![LSTM Model Prediction on whole data between year 1996-2019](image)

In the test set, we define direction-of-change as the difference between the next-period credit spread and the current spread and take the sign of this value. The formula is shown below:
predicted direction of $CS = \text{sign}(CS_{t + 1} - CS_t)$ \hspace{1cm} (17)

In the confusion matrix in Figure 15, we can see that in the test, LSTM performs extremely well. The accuracy (total true positive divided by total predicted positive) is 96.1% and the negative predictive value (total true negative divided by total predicted negative) is 81.6%.

Figure 16 compares the last two model results on the test set. Below provides some observations:

1. Although BART captures the average credit spread well, it actually performs poorly in predicting direction-of-change. That is to say, BART model is shortsighted and puts too much weight on recent observations, which undermines its predictive power on credit spread movements.
2. LSTM with features performs better than the one without features and this supports our inclusion of feature variables.
3. Although LSTM can effectively predict the direction-of-change, LSTM tends to underestimate the magnitude of the credit spread.

The accuracy below is calculated by test set correctly predicted direction of credit spread number divided by the total data number in test set.

Usually, a machine learning model is evaluated by residuals between predicted outcomes and true outcomes. Here we use

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f(x_i) - f(x_i))^2}$$ \hspace{1cm} (18)

specifically.

As Figure 17 shows, we tune the number of features and look-back period parameters in order to find the model with highest accuracy and lowest RMSE. We find that the best model has all features and look-back period of 3. The model with look-back period of 3 and no features also performs well, but has lower accuracy and higher RMSE than the model with all features included. Models with a higher look-back period than 3, begin to lose accuracy and RMSE increases. We believe that a look-back period of 3 or 4 and containing all features can both provide decent predictions.

4. Conclusion

In this paper, we used different mathematical and machine learning methods to predict the direction and magnitude of credit spread movement, and managed to get a final accuracy of almost 90%. In general we followed a trial and error process, trying to fix the model drawback by introducing new techniques. Below provides our result as well as some potential improvements.

4.1 Results

At first, we find that the credit spread follows different dynamics in different regimes by fitting the data to a Vasicek model in moving time windows. We could see that the parameters are not time homogeneous.

Then random forest model is introduced. However it cannot predict well for direction-of-change. This could be due to the labeling process, because we might lose some information of the original data.

To solve the missing value problem as well as improving the tree model performance, Bayesian Additive Regression Tree (BART) model is used and provides a good fit and prediction for next day’s credit spread. However, the direction of credit spread cannot be predicted well based on the result of our
To tackle the new problem, Long Short Term Memory (LSTM) model is chosen and does a wonderful job in predicting the direction of credit spread. Part of the reason is that the credit spread follows different patterns among different regimes. LSTM can gradually "forget" the inputs from older data in order to put more weight on current features. Besides, features mentioned in the literature act as an important role in predicting the credit spread movement and can improve model performance significantly. Overall speaking, the LSTM model with look-back period equal to 3 or 4 is the best model with both relatively low RMSE and high accuracy for out-of-sample predictions of the direction-of-change of credit spread. Adding additional lag terms does not help much in predicting the out-of-sample direction.

4.2 Potential Improvement
1. In the future, we can include some latent variables to partition different time regimes and detect regime shifts.
2. We may combine the direction prediction from LSTM and magnitude prediction from BART into one model to improve the accuracy of prediction. This process is called stacking.

Reference


