Research Challenge on the Relationship Between Momentum Trading and Options Strategies

Abstract—To investigate the relation between momentum trading and options strategies on the S&P 500, we implement a simple momentum filter known as moving-average crossover and the Black-Scholes option pricing formula to construct four portfolios over the period from January 2007 to December 2017. Several risk metrics are used to quantify return performance relative to the S&P 500 momentum strategy. Other aspects of momentum trading and the associated risks are subsequently examined.

I. INTRODUCTION

In recent years, momentum trading strategies have increased dramatically in popularity. In a general sense, trading strategies involve identifying specific market factors and making assumptions about how these factors will affect the market in the future. For momentum, these factors are based on trends in an asset’s historical prices and can be found in both cross-sectional and time-series data. Cross-sectional momentum assumes that the past information of one asset relative to another in the market will shed light on the performance of the asset in the future. High performing assets will continue to rise in value while poor performing ones will keep falling in the near future, typically from 3 to 12 month periods [1]. Similarly, time-series momentum implies that the asset’s past returns can predict its future performance, which directly contradicts the efficient market hypothesis. However, solid evidence has shown that not only does momentum exist, but the autocorrelation between the asset returns is positive at the portfolio level and negative at the individual stock level [2]. This paper explores methods of capturing time-series momentum in the S&P 500 Index.

II. IMPLEMENTATION AND METHODOLOGY

The implementation of the given portfolios was conducted over eleven years, from January 1, 2007 to December 31, 2017. The S&P 500 data used for our analysis is the daily adjusted close price of ^GSPC. Figure 1 shows the S&P 500 along with the 60- and 120-day moving average.

When calculating returns, it is difficult to establish a cost basis because the portfolios are defined in terms of a fixed holding of a certain amount of assets (i.e. one share), and sometimes the position is short. As such, the returns are calculated as the difference in dollar value of assets held at the end of two days. The returns provide an intuitive broad stroke of how an investor’s wealth would develop over time, given their participation in each portfolio.

Portfolios 2, 3, and 4 require option prices to determine returns. The options written on the S&P 500 were priced using the Black-Scholes option pricing model and the 3-month Treasury Bill.

A common method of capturing momentum is to use an indicator to signal a trend in an asset’s price. The investor should then choose a long or short position based on the information given by the signal. A different method of capturing momentum is to purchase an at-the-money straddle written on the asset. Investing in an index straddle is equivalent to buying the underlying asset when the price increases and selling when the price decreases. This paper implements and analyzes four portfolios designed to capture momentum.

Portfolios 1 and 2 are implemented using the same signals but choose different methods of being long or short. The portfolios are long when the 60-day simple moving average (SMA) price of the S&P 500 exceeds its 120-day SMA counterpart. Conversely, the portfolios are short when the 60-day SMA is below the 120-day SMA. Assuming momentum exists in the market then the idea behind these signals is simple: if the portfolio is outperforming its recent history, it is likely to continue to outperform, and vice-versa. When the market signal is bullish, portfolio 1 takes a long position on one share of the underlier while portfolio 2 takes a long position on one call option. When the market signal is bearish, portfolio 1 position is to short one share of the underlier while portfolio 2 is to long one put.

Portfolios 3 and 4 are two different methods of implementing an at-the-money straddle. In portfolio 3, the strategy is to buy a new 90-day at-the-money straddle at the close of one day and sell that straddle at the close of the next day. In Portfolio 4 a 90-day at-the-money straddle is purchased, held until expiry, and then dynamically hedged to replicate the returns of Portfolio 3.
In a Black-Scholes setting, the call price and the corresponding put price are defined in the following way. For \( t < T \),

\[
C_{BS}(S, K, \sigma, r, T, t) = S \Phi(d_+) - K \Phi(d_-) e^{-r(T-t)}
\]

\[
P_{BS}(S, K, \sigma, r, T, t) = K \Phi(-d_-) e^{-r(T-t)} - S \Phi(-d_+)
\]

where

\[
d_\pm = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{S}{K} \right) + \left( r \pm \frac{1}{2} \sigma^2 \right) (T-t) \right] \]

\( S \) = the current trading price of the underlying asset in unit of currency
\( K \) = the strike price of the option in unit of currency
\( \sigma \) = the annualized standard deviation of the returns of the underlying assets
\( r \) = the annualized risk-free rate of return
\( T \) = the time to maturity, in years
\( t \) = the current time, in years

\section*{III. ANALYSIS OF PROPOSED STRATEGIES}

An important result that is common across all strategies, is the nature of their performance relative to the S&P 500. The onset of the recession provides a clear illustration of how portfolios 2, 3, and 4 capitalize off a market environment that sustains directional movement. This was evidenced by a large divergence in returns with the S&P 500, which due to exhibiting strong downward pressure, provided clear moving average indication for the portfolios to adhere to. Furthermore, as the market went through a period lacking any strong directional movement (around 2015), the strategies all exhibited weaker performance characterized by a notable downturn. This is not unexpected however, as by definition, momentum strategies hinge on the presence of strong market trends preceded by a general influx of volatility. Further analysis of each portfolio using the 90-day moving average volatility is as follows.

\subsection*{A. Portfolio 1:}

Intended to capture the momentum exhibited in the market, portfolio 1 takes a long or short position on one unit of index depending on the relative performance of the 60-day and the 120-day SMA of the index:

- long one unit of the index, if the 60-day SMA was greater than the 120-day SMA
- short one unit of the index, otherwise

By nature, the momentum strategy seeks to make profits during consistent upward or downward trends. As expected,
portfolio 1 performed with significant gains during the long sustained trend at the end of 2012 to early 2015, at the end of which the value of the portfolio reached $1182.52. However, the portfolio suffered significant losses during a period of high volatility and no discernible trend from the middle of 2015 to the middle of 2016, resulting in the portfolio value getting reduced by half.

Implementation Results (01/01/2007 – 12/31/2017)

<table>
<thead>
<tr>
<th>Summary Statistics for Portfolio Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar Returns</td>
</tr>
<tr>
<td>Portfolio 1</td>
</tr>
<tr>
<td>Min</td>
</tr>
<tr>
<td>Max</td>
</tr>
<tr>
<td>1. Quartile</td>
</tr>
<tr>
<td>3. Quartile</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Variance</td>
</tr>
<tr>
<td>Standard Dev</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Kurtosis</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk Metrics for Portfolio Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar Returns</td>
</tr>
<tr>
<td>Portfolio 1</td>
</tr>
<tr>
<td>VaR0.95</td>
</tr>
<tr>
<td>CVaR0.95</td>
</tr>
<tr>
<td>MDD</td>
</tr>
<tr>
<td>Duration of MDD(days)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Drawdown for Portfolio Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMD</td>
</tr>
<tr>
<td>Portfolio 1</td>
</tr>
<tr>
<td>Trough</td>
</tr>
<tr>
<td>Cumulative Returns</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Portfolio 1 involves taking substantial short positions. That is, over the period from January 1, 2007 to December 31, 2017, the portfolio will require holding a short position about 25% of the time. All else equal, an investor abiding to a short position will be more exposed to potential financial pain, illustrating that this strategy can burden the investor with significant risk.

While the variance of portfolio 1 and that of the benchmark S&P 500 are almost identical, the average return of portfolio 1 is noticeably smaller than that of the S&P 500 as evidenced by the provided risk metrics.

B. Portfolio 2:

Portfolio 2 is structured as follows:
- long one call option, if the 60-day SMA was greater than the 120-day SMA
- short one put option, otherwise

As expected, similar to portfolio 1, portfolio 2 suffers significant loss during periods of high volatility and periods in which no discernible trends are detected. The largest peak-to-trough decline in the value of the portfolio occurred during the same period. However, the standard deviation is cut in half in exchange for lower returns. Similar observations can be made for the risk metrics.

C. Portfolio 3:

Portfolio 3 involves constantly holding a long position on an ATM straddle - long an ATM call and put on the same underlying asset with the same maturity and the same strike price. An ATM straddle capitalizes on the volatility of the market, regardless of the direction of the movement of the underlying asset. By nature, a straddle has limited loss, limited to the premium paid, and a theoretically unlimited profit.

Compared to portfolio 1 and 2, portfolio 3 has smaller returns, but smaller standard deviation. The distribution of return for portfolio 3 is right-skewed. As expected, portfolio 3 outperformed portfolio 1 during the crisis, but the greatest decline in value is also during this same time, during which the portfolio lost half of its value.
D. Portfolio 4:

A dynamically hedged strategy involves rebalancing the current position according to market changes. Assuming that for an infinitesimal change in the underlying market parameters, the value of equivalent assets will change in the same way, replicating an ATM straddle is boiled down to the construction of portfolios with equivalent partial derivatives with respect to market parameters of interest - the Greeks.

Portfolio 4 requires holding a straddle until maturity and subsequently hedging the position on a daily basis so it is equivalent to an ATM straddle each day. Again, replicating an ATM straddle is equivalent to constructing a portfolio in which the Greeks of the portfolio matches that of an ATM, 90-day straddle on the same underlying for any given day using shares of the underlying assets.

The value of an ATM straddle heavily depends on the movement of the price of the underlying asset; as the asset price increases, the call option will become in-the-money while the put option falls out-of-money. A solid directional movement in the price of the underlying assets will determine whether the call or the put will be exercised at maturity. Accordingly, for an increase in the value of the underlying asset, the Delta ($\Delta$), or the change of the option’s value with respect to a change in the asset price, increases as the call option runs in-the-money while the $\Delta$ of a put decreases as the option becomes out-of-money. Close to maturity, if the call option is to be exercised, the $\Delta$ will grow close to 1. On the other hand, if the put option is to be exercised, the $\Delta$ will approach -1.

As both call and put are convex functions of the price of the underlying asset, the degree of convexity, or the second derivative of the portfolio value with respect to the price of the underlying asset determines the magnitude of the loss or the gain as the price of the underlying asset moves. Therefore, a $\Delta$ - $\Gamma$ - equivalent strategy will guarantee that as the price of the underlying asset moves, the loss or the gain of the replicated portfolio will be similar to that of an ATM straddle, ceteris paribus.

Let $Z_t$: the value of a $\Delta$-$\Gamma$-equivalent portfolio composed of a call $C_t$, a put $P_t$, some shares of the underlying asset $S_t$ needed to equate the $\Delta_t$ and some shares of an additional asset with non-zero well-defined second derivative $\beta_t$ at time $t$

$\Delta_t$: the $\Delta$ of an ATM straddle at time $t$

$I_t$ : the $\Gamma$ of an ATM straddle at time $t$

$A_t$ : the price of the additional asset at time $t$

$$Z_t = C_t + P_t + \beta_t A_t + \alpha_t S_t$$

$$\frac{\partial Z_t}{\partial S_t} = \frac{\partial C_t}{\partial S_t} + \frac{\partial P_t}{\partial S_t} + \beta_t \frac{\partial A_t}{\partial S_t} + \alpha_t$$

$$\frac{\partial^2 Z_t}{\partial S_t^2} = \frac{\partial^2 C_t}{\partial S_t^2} + \frac{\partial^2 P_t}{\partial S_t^2} + \beta_t \frac{\partial^2 A_t}{\partial S_t^2} + \alpha_t$$

Set $\frac{\partial^2 Z_t}{\partial S_t^2} = I_t$, 

$$I_t = \frac{\partial^2 C_t}{\partial S_t^2} + \frac{\partial^2 P_t}{\partial S_t^2} + \beta_t \frac{\partial^2 A_t}{\partial S_t^2}$$

The combination of a straddle held until expiration, of shares of underlying assets, and of contracts of additional assets will be identical to holding an ATM straddle at all time in terms of the $\Delta_t$ and $I_t$. On expiration day, the straddle becomes worthless, the position on the underlying and the additional assets is closed and a new straddle is purchased.

Clearly, the $\Delta$ - $\Gamma$ - equivalent strategy underperforms portfolio 3, because of the accumulated decay in time value of money. In theory, a $\Delta$-$\Gamma$-$\Theta$ equivalent strategy would have performed better with a fifth asset introduced in the model to match the Theta ($\Theta$) or the change with respect to time.

Note that the return stream on portfolio 4 is similar to that of portfolio 3 by nature, but exhibits returns of much smaller magnitude, providing evidence that extensive hedging with the Greeks may not be sustainable over the long run. See summary statistics for further support.

E. Volatility Estimates:

Figure 5 shows the performance of all four portfolios using the 90-day volatility implied by VXV. As expected, this measure significantly diminished portfolio returns in all portfolios that require option pricing. The return reduction is most pronounced in Portfolios 2 and 3, most likely because a straddle is essentially a bet that market volatility will be more than the market forecast. This confirms our assertion that VXV is an overestimate of volatility.

![Figure 5: Cumulative Dollar Return of 4 Portfolios using the 90-day volatility implied by VXV](image-url)
Figure 6 shows the performance of all four portfolios using the 90-day volatility implied by VXV adjusted to match the mean of the realized distribution. What is most noticeable about this chart when compared with the 90-day trailing volatility is the strong performance of Portfolio 4 after 2014. This is because the rate at which VXV overestimates realized volatility is not constant over time. In 2014 and 2015 the overestimate is much smaller (about 1 percentage point) than the rest of the sample (about 4 percentage points) and so the mean adjusted VXV drives strong returns in that time frame. This underscores the difficulty of trying to determine how much VXV is actually overestimating volatility.

IV. MONTE CARLO ANALYSIS

An important component of portfolio investment analysis is developing a systematic approach to forward-looking risk management. To establish this in practice, realistic future scenarios of value-driving variables need to be produced as a function of an underlying probability distribution and modeling assumptions. Thus, the framework for the model was built on the assumption that the future prices of the S&P 500 can be accurately modeled with a geometric Brownian motion with constant drift and volatility. From the simulated data, Black-Scholes option prices are computed, enabling a derived simulation of the various trading strategies to be produced.

Let $S_t$, be the price of the underlying asset at time $t$.

$$S_t = S_0 e^{\mu t + \frac{1}{2} \sigma^2 t}$$

where

- $W_t$ is a standard Brownian motion $W_t \sim N(0, t)$
- $\mu$ is the constant historical mean of return
- $\sigma$ is the constant historical volatility of return

Equivalently in discretized form,

$$S_t = S_0 e^{\mu dt + \frac{1}{2} \sigma \sqrt{t} Z}$$

where $Z \sim N(0,1)$

The model simulates 5,000 cumulative dollar return outcomes for portfolio 1, 2 and 3 over a 252-trading-day period. Using data from January 1, 2015 - December 31, 2017, annualized mean and volatility of the daily log S&P 500 returns were calculated and used as the geometric Brownian motion drift and diffusion terms respectively. Then computing daily option prices along each simulated S&P 500 path, a constant 3-month risk-free rate of 1.7% was assumed along with a trailing 63-trading-day volatility for that path’s corresponding S&P 500 arithmetic return. From this, a fairly comprehensive view of possible portfolio outcomes is given, allowing for the deduction of risk metrics and a general sense of what to expect financially for the simulated time horizon.

As evidenced by the resulting simulated future scenarios, path dispersion decreases from portfolio 1 to 3. That is, each Monte Carlo analysis reflects of the general risk reward trade-off present in our historical implementation. We further represent this by computing the 95% significance level, one-day value-at-risk and expected shortfall for strategy participation.

### IV. RISK METRICS FOR MONTE CARLO SIMULATION

<table>
<thead>
<tr>
<th>Dollar Returns</th>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR$_{0.95}$</td>
<td>-40.28</td>
<td>-20.44</td>
<td>-7.45</td>
</tr>
<tr>
<td>CVaR$_{0.95}$</td>
<td>-50.64</td>
<td>-26.35</td>
<td>-10.69</td>
</tr>
</tbody>
</table>

An important caveat regarding the presented simulation and the loss probability for portfolio 3 is that in order for the model to yield negative cumulative outcomes by year end 2018, the geometric Brownian motion model must be parameterized by a sufficiently low level of volatility (at most 0.075 compared to the ~0.10 calculated in the presented models), misleadingly suggesting that the straddle is riskless. Intuitively, as straddles capitalize on volatility, a low amount of market movement in either direction is sufficient for the strategy to capture some magnitude of return. As such, it is more realistic to use value-at-risk on a shorter time frame, in which portfolio 3 does indeed exhibit losses - reflecting that the strategy is not in fact riskless.

From the vantage point of an investor evaluating potential momentum trading strategies to engage in at the onset of 2018, it is important to note that while the above metrics are useful for risk management, limitations of the model must be kept in consideration. First, the market environment from which the underlying stock model parameters were calculated, was one of strong growth for the S&P 500. This could likely serve to overestimate the positive drift each path exhibits, simulating a future with a higher growth rate than would likely be observed. Additionally, a pitfall of using the geometric Brownian motion model is the fact that it draws from a standard normal distribution in its generation of the Brownian motion terms that apply random shocks to the model’s constant variance. This is
an inadequacy as it yields a lighter tailed distribution of returns relative to what’s realized historically. Thus the dispersion and the probability of realizing negative outcomes are likely understated.

Lastly, as a hedged version of portfolio 3, portfolio 4 exhibited even less variance, dollar return and an eventual value decay, implying a low likelihood of divergence from this observed behavior. This relatively poor performance coupled with the mounting transaction costs, render computationally exhaustive risk management techniques including monte carlo analysis, overzealous and likely misrepresentative for risk management purposes.

**Monte Carlo Results (01/01/2018 – 12/31/2018)**

![Figure 7: Monte Carlo Simulation of Portfolio 1](image)

![Figure 8: Monte Carlo Simulation of Portfolio 2](image)

![V. INFLUENCE OF TRADING SIGNALS](image)

The analysis conducted was based on the signals given by comparing the 120-day SMA and the 60-day SMA. However, had the signals been different, the results would have changed drastically.

![Figure 10: Performance of all Portfolios using 30-days SMA and 60-day SMA](image)

Had the momentum been set-up using the signals produced by the 30-day SMA and the 60-day SMA, portfolio 1 would have incurred a huge loss. However, had the momentum been set based on the signals produced by the 90-day and 120-day, portfolio 1 would have outperformed the market index. Notice that in either case, portfolio 2 also shares the same fate, but to a lesser extent. As such this is a brief illustration of how choice of indicator can heavily influence momentum trading results.
V. ALTERNATIVE INDICATOR SUMMARY STATISTICS

<table>
<thead>
<tr>
<th>Dollar Returns</th>
<th>Portfolio 1 SMA30/MA60</th>
<th>Portfolio 2 SMA30/MA60</th>
<th>Portfolio 1 SMA90/MA120</th>
<th>Portfolio 2 SMA90/MA120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-104.13</td>
<td>-27.77</td>
<td>-104.13</td>
<td>-27.77</td>
</tr>
<tr>
<td>Max</td>
<td>106.85</td>
<td>72.38</td>
<td>106.85</td>
<td>72.38</td>
</tr>
<tr>
<td>1. Quartile</td>
<td>-7.89</td>
<td>-4.39</td>
<td>-7.03</td>
<td>-4.15</td>
</tr>
<tr>
<td>3. Quartile</td>
<td>8.31</td>
<td>3.88</td>
<td>8.80</td>
<td>4.23</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.12</td>
<td>0.06</td>
<td>0.46</td>
<td>0.35</td>
</tr>
<tr>
<td>Median</td>
<td>0.22</td>
<td>-0.46</td>
<td>0.64</td>
<td>-0.31</td>
</tr>
<tr>
<td>Variance</td>
<td>271.93</td>
<td>64.02</td>
<td>271.73</td>
<td>67.52</td>
</tr>
<tr>
<td>Stdev</td>
<td>16.49</td>
<td>8.00</td>
<td>16.48</td>
<td>8.21</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.20</td>
<td>0.96</td>
<td>-0.09</td>
<td>1.08</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.83</td>
<td>5.14</td>
<td>3.85</td>
<td>5.26</td>
</tr>
</tbody>
</table>

VI. MEAN-VARIANCE OPTIMIZATION

A classic financial outlook in the context of effective portfolio management is to optimize the tradeoff between the two parameters, mean and variance. In general, a rational investor would only deem a portfolio worth investing if it resided on the upper half of the efficient frontier, i.e. the set of portfolios with the optimal expected return for a given level of volatility or lowest magnitude of volatility when imposing a fixed threshold for acceptable return. Because of these clear risk management implications, it is relevant to consider the plausibility of imposing this two-parameter optimization in the context of portfolio allocation and investment decision making.

Specific to the proposed strategies, mean-variance optimization approach would perform poorly in terms of producing reliable results. Although the distributions of return from both portfolio 1 and the S&P 500 are almost symmetrical, their returns are linearly correlated which, in this case, lessens the effectiveness of the overall purpose of diversification. The returns from portfolios 2, 3, and 4, however, are not normally distributed (see Figure 12); in addition to exhibiting the right-skewed and heavy tailed behaviors (see Table I), options from these portfolios are time-dependent. As the result, their estimated variances become ineffective as a useful measure of risk [3].

Figure 12: Distribution of Returns of all Portfolios

Nevertheless, taking into account the inherent financial leverage provided by options on S&P 500, it is possible to combine all the proposed portfolios and apply a more advanced optimization technique which falls beyond the scope of this paper.

VII. CONCLUSION

The net result of researching the behavior and structure of the proposed portfolios was an ability to gauge the effectiveness of using Black-Scholes options in the context of momentum trading and replication. This offered perspective of how return magnitude is highly subject to the market dynamics through which daily trading action is taking place. Namely, a sustained period of low growth or decay exposes the pitfall of the proposed portfolios’ perpetual dependence on the presence of momentum.

This leads us to a concept that is fundamental regarding the idea of risk in the practice of conditioning trading decisions on a quantity, or an indicator, assumed to be linked to current market direction. The concept is indication quality. That when trading off momentum, the definition of risk needs to be extended beyond volatility to include the likelihood that the indicator mistimes or misreads the market’s momentum. That when consecutive gains or drops in relevant prices are in fact taking place in the market, it is possible that the indicator will inform the strategy incorrectly, or not condition the strategy with this information until either the directional movement has progressed significantly, already passed, or not yet taken place. Thus at any given point in time, risk is not so much comprised of the market position itself, but of the incoming indicator data
that will cause inconsistencies in market sentiment between the strategy itself and the market’s underlying direction.

Much of this indicator risk is systematic, however as was illustrated in the timeframe of our portfolio analysis, it can be reduced or eliminated by acting on market direction through dealing in options. Hence, the motivation for deploying three replication methods built around transacting in options rather than the index itself.

Finally, this paper at its core drives at how portfolio analytics is essential to understanding the nature and influence of capturing time-series momentum in the S&P 500 index. That when structuring a portfolio intended to capitalize off market direction, the only class of strategies immune to indicator risk are those with bidirectional market exposure accompanied by low upside potential and a strong dependence on volatility. Otherwise, strategy prosperity can be viewed as a function of its interaction with non-directional environments as well as its ability to consistently produce an accurate market response. As such these qualities provide the important foundation in the relationship between signal capitalization, hedging and external market factors.

REFERENCES

