Variable Volatility and Financial Failure

Peter Carr

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- I'd also like to thank my former MS colleague Harry Mendell for the quant joke on the next slide.

Quant Question

What does the middle initial "B." in:

Benoit B. Mandelbrot

stand for?

Quant Answer

The middle initial "B." in:

Benoit B. Mandelbrot

stands for:

Benoit B. Mandelbrot.

All Joking Aside

- Mandelbrot emphasized self-similarity in both nature and finance.
- Many of the financial formulas you know and all of the financial formulas shown in this presentation are a consequence of an exact form of self-similarity called scale invariance.

Derivatives and Default

- On October 14 1997, the Royal Swedish Academy of Sciences announced the award of the Nobel prize in Economic Sciences to Professors Merton and Scholes for a new method to determine the value of derivatives.
- In the press release, the academy observed that:

  > *The value of the stock, preferred shares, loans, and other debt instruments in a firm depends on the overall value of the firm in essentially the same way as the value of a stock option depends on the price of the underlying stock. The laureates had already observed this in their articles published in 1973, thereby laying the foundation for a unified theory of the valuation of corporate liabilities.*

- This approach to modeling corporate default is known as a structural model.

Parametric & Non-parametric Structural Models

- Structural models are usually parametric. For example, Merton (1974) uses a single parameter to describe the volatility of the firm's assets.
- A drawback of the parametric approach to structural models is that it fails to recognize the degree of flexibility that management has in setting the assets' volatility. In principle, management can choose a different volatility level at every possible value in $[0, 1)$ for (risk-neutral) default probability (RNDP).
- A second drawback of parametric structural models is that they need not be consistent with the rich information content embedded in the equity option smile.
- In this talk, we develop a non-parametric approach to structural models. The volatility of the firm's assets will be a function of (just) RNDP, which is calibrated to a given equity option smile.

Calibrating Merton's Parametric Structural Model

- Assuming that the firm's asset value follows Geometric Brownian Motion (GBM) with known volatility, Merton (1974) gives closed form formulas for (risk-neutral) default probability (RNDP) and stock price as functions of the firm's asset value and volatility and the firm's promised debt payment.

- When the firm's asset value and volatility are ex ante unknown, they can be backed out of the market price of the stock and a stock option. This calibration requires that the promised debt payment be known.

- In the end, one can numerically relate the firm's RNDP and distance-to-default to the market price of the stock and a stock option.

Applying Merton's Parametric Structural Model to Banks

- When compared to other firms, banks employ higher leverage, but they also have greater flexibility in their operations.
- When applied to banks, the standard parametric Merton model tends to give higher default probabilities than has been realized. This lead KMV and others to use a non-parametric version of Merton using historical data on defaults to determine the function relating (real world) default probability to distance to default.
- This talk presents an alternative non-parametric specification of Merton, which bypasses the need to collect historical default data or estimate distance to default. Instead, stock and stock option prices of all strikes are used to calculate risk-neutral default probabilities and distance-to-default.

Choosing & Calibrating a Non-Parametric Structural Model

- By assuming that asset volatility is a function of (just) distance to default, we follow Merton in giving closed form formulas for (risk-neutral) default probability (RNDP) and stock price as function(al)'s of the firm's asset value, the asset volatility function, and the firm's promised debt payment.

- When the firm's asset value and the firm's asset volatility function are ex ante unknown, we will show how to *analytically* calculate RNDP and distance-to-default as functionals of the stock price and a local volatility smile derived from equity option prices.

- In contrast to Merton, our formulas for RNDP and distance-to-default are both *independent* of the promised debt payment L. Instead, they just depend on the market data described above and the time to the debt maturity $M - t$.

- We can also numerically determine the option market's beliefs concerning the relationship between asset volatility and RNDP/distance-to-default.

Real-Valued Assets

- Recall Merton's classical setup. The value A of the firm's assets is given by a geometric Brownian motion (GBM) G_1. The firm has a single issue of zero coupon debt with face value $L > 0$ and maturity date $M > 0$. The shareholders receive $(G_{1M} - L)^+$ at time M.

- Now suppose at time 0 that the firm shorts a second asset whose price is an independent GBM G_2. Suppose that there are no margin requirements or short sales constraints so that the firm does not need to supply funds if G_2 skyrockets. In this case, default can still only occur at M and it occurs just if $G_{1M} - G_{2M} < L$. In this case, shareholders get nothing.

- If $G_{1M} - G_{2M} \geq L$, then shareholders receive $G_{1M} - G_{2M} - L$ at time M, so at any time $t \in [0, M]$, shareholders own a call on a spread of GBM's.

- In this talk, we think of the asset value more generally as the difference in value of long and short positions in risky assets. Hence, the value of the assets will be a real-valued stochastic process.

Distance to Default

- Let $A_t \in \mathbb{R}$ denote the value of the firm's assets at some time $t \geq 0$.
- Let $L > 0$ denote the face value of the firm's zero coupon debt issue.
- Let $M > 0$ be the maturity date of the debt.
- We define distance to default at time t by $z_t = \frac{A_t - L}{\sqrt{M-t}}$ for $t \in [0, M)$. This is the number of standard deviations of A_M that the current asset value A_t exceeds face value L, if the asset value process were Brownian motion started at A_t.
- Notice that distance to default will almost surely diverge to $\pm\infty$ as $t \uparrow M$.
- The firm defaults at $t = M$ if and only if $z_M = -\infty$. At any prior time $t \in (0, M)$, one expects that the risk-neutral default probability (RNDP) is declining in distance to default.

Risk-Neutral Asset Value Dynamics

- Suppose zero interest rates and zero dividends to shareholders.
- Assuming no arbitrage, there exists a probability measure $\mathbb{Q}$ such that the values of all non-dividend paying assets evolve as a martingale.
- Recall that A_t denotes the value of the firm's assets and that L denotes the promised payment to bondholders at maturity date M.
- The $\mathbb{Q}$ dynamics of A are given by:

$$dA_t = \eta(z_t) dW_t, \qquad t \in [0, M),$$

where W is a $\mathbb{Q}$ standard Brownian motion and recall that $z_t = \frac{A_t - L}{\sqrt{M-t}}$ describes distance to default.

- Under $\mathbb{Q}$, the asset value is a continuous martingale whose normal volatility η just depends on distance to default z.

Restrictions on Normal Volatility Function

- Recall our assumption on the $\mathbb{Q}$ dynamics of asset value A:

$$dA_t = \eta(z_t)dW_t, \qquad t \in [0, M),$$

where recall $z_t = \frac{A_t - L}{\sqrt{M-t}}$ describes distance to default.

- We assume that the volatility function $\eta(z), z \in \mathbb{R}$ is positive and bounded away from zero.

- Recall that as the debt nears its maturity, distance to default diverges almost surely to either positive infinity or negative infinity. As a result, we also assume that the dollar volatility of the assets asymptotes to a finite nonzero constant as $|z| \to \infty$.

- An example of such a volatility function is the one in Bachelier's model, $\eta(z) = \eta > 0$. A counterexample is $\eta(z) = z$.

Scale Invariance

- Recall our assumption on the $\mathbb{Q}$ dynamics of asset value A:

$$dA_t = \eta \left(\frac{A_t - L}{\sqrt{M - t}} \right) dW_t, \qquad t \in [0, M).$$

- Consider dilating the spatial variables A and L by a positive constant λ:

$$A \to \lambda A \qquad L \to \lambda L$$

while simultaneously dilating the time variables t and M by λ^2:

$$t \to \lambda^2 t \qquad M \to \lambda^2 M.$$

- The Brownian scaling property implies that the new SDE has the same statistical laws as the original one. This invariance allows us to treat space and time as different versions of the single variable $z_t = \frac{A_t - L}{\sqrt{M-t}}$ that we call distance to default.

Remainder of the Talk

- The remainder of this talk splits into two halves.
- Part 1: Evaluating the failure probability and stock price when asset value A, debt level L, and asset volatility function $\eta(z)$ are all known.
- Part 2: Evaluating the failure probability and distance to default when asset value A, debt level L, and asset volatility function $\eta(z)$ are all unknown.
- In Part 2, we use the market price of the firm's stock along with the market price of stock options to determine the implied failure probability and the implied distance to default.

Part 1: Known Asset Volatility Function

- In this part of the talk, we assume that one can directly observe the asset value A and the promised debt payment L.
- Throughout this talk, we will be assuming that the debt maturity date M is known.
- As a result, the distance to default $z = \frac{A-L}{\sqrt{M-t}}$ is directly observed in this part of the talk.
- Our objective is to determine the risk-neutral default probability, RNDP, and the equity value as functions of asset value A, time t, debt level L, and maturity date M.

Calculating Risk-Neutral Default Probability (RNDP)

- Recall that the firm can only default at time M when the payment L is due.
- Let $F(A, t; L, M)$ be the firm's (risk-neutral) failure probability, considered as a function of the asset value $A \in \mathbb{R}$ and calendar time $t \in [0, M)$.
- In our model, we show that the failure probability at time t depends on its four arguments A_t, t, L, and M only through the distance to default variable $z_t = \frac{A_t - L}{\sqrt{M-t}}$.
- In fact, the firm's failure probability is given by a closed form formula:

$$F(A, t; L, M) = 1 - \frac{1}{b_f} \int_{-\infty}^{z} e^{-\int_0^y \frac{x}{\eta^2(x)} dx} dy, \qquad z \in \mathbb{R},$$

where the positive normalizing constant b_f is given by:

$$b_f = \int_{-\infty}^{\infty} e^{-\int_0^y \frac{x}{\eta^2(x)} dx} dy.$$

Asset Vol & Risk-Neutral Default Probability (RNDP)

- Recall that our assumption that asset volatility depends only on $z = \frac{A-L}{\sqrt{M-t}}$:

$$dA_t = \eta(z_t)dW_t, \qquad t \in [0, M),$$

has lead to a closed form formula for the RNDP at time $t \in [0, M)$:

$$F(A, t; L, M) = 1 - \frac{1}{b_f} \int_{-\infty}^{z} e^{-\int_0^y \frac{x}{\eta^2(x)} dx} dy, \qquad z \in \mathbb{R},$$

where b_f is a positive constant.

- This formula indicates that the failure probability $F(A, t; L, M)$ is a decreasing function $f(z)$ of just the distance to default $z = \frac{A-L}{\sqrt{M-t}}$. Let $\tilde{z}(F)$ be the inverse function. Then, distance to default $\tilde{z}(F)$ is decreasing in the failure probability F and depends only on this variable.

- It follows that the assets' $ volatility just depends on the failure probability:

$$dA_t = \tilde{\alpha}(F_t)dW_t, \qquad t \in [0, M), \text{ where } \tilde{\alpha}(F) = \eta(\tilde{z}(F)).$$

Valuing Equity When Asset Vol Just Depends on RNDP

- Let $S(A,t;L,M)$ be the stock value function in our model:

$$S(A,t;L,M) = E^{\mathbb{Q}}[(A_M - L)^+ | A_t = A], \qquad A \in \mathbb{R}, t \in [0,M).$$

- We show that the normalized stock price $s_t = \frac{S_t}{\sqrt{M-t}}$ depends on the four variables A_t, t, L, and M only through the distance to default variable $z_t = \frac{A_t - L}{\sqrt{M-t}}$.

- In fact, the firm's equity value is given by the following closed form formula:

$$S(A,t;L,M) = \frac{1}{b_s}\left[\sqrt{M-t}\,e^{-\int\limits_0^z \frac{x}{\eta^2(x)}dx} + (A-L)\int\limits_{-\infty}^{z} \frac{e^{-\int\limits_0^y \frac{x}{\eta^2(x)}dx}}{\eta^2(y)}dy\right],$$

where $z = \frac{A-L}{\sqrt{M-t}}$ and $b_s = \int\limits_{-\infty}^{\infty} \frac{e^{-\int\limits_0^y \frac{x}{\eta^2(x)}dx}}{\eta^2(y)}dy$.

Asset Volatility and Normalized Stock Price

- Recall that our assumption that asset volatility depends only on $z = \frac{A-L}{\sqrt{M-t}}$:

$$dAt = \eta(z_t)dW_t, \qquad t \in [0, M),$$

  implies that the normalized stock price $s_t = \frac{S_t}{\sqrt{M-t}}$ is a function $s(z)$ of just the distance to default process $z_t = \frac{A_t-L}{\sqrt{M-t}}$.

- Since $s(z)$ is increasing in z, it follows that there exists a function $z(s)$ which is increasing in s.

- It follows that the assets' dollar volatility just depends on the normalized stock price in our class of models:

$$dA_t = \alpha(s_t)dW_t, \qquad t \in [0, M),$$

  where $s_t = s\left(\frac{A_t-L}{\sqrt{M-t}}\right)$ is the normalized stock price at time t and $\alpha(s) \equiv \eta(z(s))$ is the assets' dollar volatility, written as a function of the normalized stock price.

Local Volatility for Stock Price

- Recall that in our class of models, the assets' dollar volatility just depends on the normalized stock price $s_t = \frac{S_t}{\sqrt{M-t}}$:

$$dA_t = \alpha(s_t)dW_t, \qquad t \in [0, M).$$

- It turns out that the stock's delta, $\frac{\partial S}{\partial A}$, depends on the four variables A_t, t, L, and M only through the distance to default variable $z_t = \frac{A_t - L}{\sqrt{M-t}}$.

- Letting $p(z)$ be the stock's delta w.r.t. assets, Itô's formula implies:

$$dS_t = p(z(s_t))\alpha(s_t)dW_t, \qquad t \in [0, M),$$

  where recall that $z(s)$ is the inverse of the normalized stock pricing function $s(z)$.

- Since the stock's volatility just depends on the stock's price and time, the stock price dynamics are described by a (special case of a) local vol model.

Stock Price and Failure Probability

- Recall that the failure probability F and the normalized stock price $s = \frac{S}{\sqrt{M-t}}$ are each just a monotonic function of distance to default $z = \frac{A-L}{\sqrt{M-t}}$.

- Since the normalized stock price s is an increasing function of z, it follows that there exists an inverse function $z(s)$, which is increasing in s.

- Since the failure probability F is a decreasing function of only z, it follows that the failure probability F is also some decreasing function $\phi(s) = f(z(s))$ of only s.

- When the asset volatility is a specified function of z, the inverse functions $z(s)$ and $\phi(s)$ can only be determined numerically.

- However, when stock volatility is a specified function of normalized stock price, then we have closed form formulas for the inverse functions $z(s)$ and $\phi(s)$, which we present in the next part of the talk.

Summarizing Part 1

- The standard Merton model assumes that the normal volatility η of a firm's assets is positively proportional to the value A of the firm's assets i.e. $\eta_t = \sigma A_t, \sigma > 0$.
- We instead assume that normal volatility of a firm's assets is a function of the risk-neutral default probability (RNDP).
- By modeling the function $\eta(z)$ relating asset volatility η to distance to default $z = \frac{A-L}{\sqrt{M-t}}$, we were able to give a closed form formula for both the RNDP and for the value of the firm's equity.

Summarizing Part 1 (Cond)

- To our knowledge, our explicit valuation formulae for RNDP and equity value are the first to be derived when asset volatility is free to be any function, rather than described by a particular functional form with a small number of free parameters.

- When the asset value A and the debt level L are not directly observed, we will show that the flexibility of our asset volatility specification allows the distance to default $z = \frac{A-L}{\sqrt{M-t}}$ to be determined from the market's stock price and the market prices of co-terminal calls written on the stock.

- To determine this implied distance to default, one does not specify the asset volatility function. Instead, the equity volatility is taken as a given function of the normalized stock price. This local volatility smile is obtained from stock option prices.

- In the next part of the talk, we will show that we can in fact explicitly relate both distance to default and RNDP to this data.

Part 2: Calibrating to Stock and Stock Option Prices

- In this part, we continue to assume that the debt maturity date M is known. However, we now suppose that the debt level L and the asset value A cannot be observed. We also assume that the asset volatility function $\eta(z)$ is not known ex ante.
- To make up for this informational shortfall, we assume that one can observe the initial market price of the stock S_0.
- We also assume that there exists a liquid market in calls written on the stock that mature before the debt. We assume that one can directly observe the initial market prices $C_0(K)$ of these calls for all strikes $K > 0$ at some fixed maturity date $T \in [0, M]$.

Obtaining a Local Vol Smile from Option Prices

- Recall that $C_0(K, T)$ is the market price at time $t = 0$ of a call written on the stock struck at some $K > 0$ and maturing at some $T \in [0, M]$.
- At its maturity date T, the call pays $[S(A_T, T; L, M) - K]^+$ dollars to its holder.
- We assume that market call prices $C_0(K, T)$ and infinitessimal calendar spreads $\frac{\partial}{\partial T} C_0(K, T)$ are both observable for a continuum of strikes $K > 0$ at the single maturity date $T \in [0, M]$.
- From Dupire (1996), one can observe the risk-neutral mean of the instantaneous variance rate of equity at T, conditional on the event $S_T = K$. i.e. $\frac{2 \frac{\partial}{\partial T} C_0(K,T)}{\frac{\partial^2}{\partial K^2} C_0(K,T)} =$

$$E^{\mathbb{Q}} \left[\left(\frac{\partial}{\partial A} S(A_T, T; L, M) \right)^2 \times \eta^2 \left(\frac{A_T - L}{\sqrt{M - T}} \right) \middle| S(A_T, T; L, M) = K \right].$$

Obtaining a Local Vol Smile from Option Prices

- For the fixed call maturity date T, let:

$$\hat{a}^2(K) \equiv \frac{2\frac{\partial}{\partial T}C_0(K,T)}{\frac{\partial^2}{\partial K^2}C_0(K,T)}, \qquad K > 0,$$

  be the directly observed local variance rate, as a function of absolute strike.

- Defining normalized strike price by $k \equiv \frac{K}{\sqrt{M-T}}$, one can instead relate the directly observed local variance rate to this normalized strike:

$$a^2(k) \equiv \hat{a}^2(k\sqrt{M-T}) = \left.\frac{2\frac{\partial}{\partial T}C_0(K,T)}{\frac{\partial^2}{\partial K^2}C_0(K,T)}\right|_{K=k\sqrt{M-T}}, \qquad k > 0.$$

- Our formulas for failure probability and for distance to default will depend on the function $a^2(s)$. This is the local variance rate of the stock, $\frac{(dS_t)^2}{dt}$ considered as a function of the normalized stock price $s_t = \frac{S_t}{\sqrt{M-t}}$. Given both $C_0(K,T), K > 0$ and $\frac{\partial}{\partial T}C_0(K,T), K > 0$, this function is known.

RNDP as a Function of Stock Price & Time to Maturity

- In our setting, the failure probability is given by a closed form formula:

$$F(A(S,t;L,M),t;L,M) = 1 - \frac{1}{b_\phi} \int_0^{\frac{S}{\sqrt{M-t}}} e^{-\int_0^y \frac{z}{a^2(z)}dz} dy, \qquad S > 0, t \in [0,T),$$

  where the positive normalizing constant b_ϕ is given by:

$$b_\phi = \int_0^\infty e^{-\int_0^y \frac{z}{a^2(z)}dz} dy.$$

- Notice that the RHS does not depend on the time t asset value A or on the face value L of the debt. Instead, the failure probability only depends on the time t stock price S, the time to the debt maturity $M-t$, and the local volatility function $a^2(k) \equiv \left.\frac{2\frac{\partial}{\partial T}C_0(K,T)}{\frac{\partial^2}{\partial K^2}C_0(K,T)}\right|_{K=k\sqrt{M-T}}$, $k>0$ determined from the initial call prices. These variables represent sufficient statistics for the determination of the RNDP.

Distance to Default as a Function of Stock Price & Time

- We now switch attention to the problem of inverting the normalized stock pricing function $s(z)$. Let $z(s), s > 0$ denote the desired inverse.

- We assume that $\lim_{x \downarrow 0} \frac{a(x)}{x^p} \leq c$, for some $p \geq 1$ and some constant $c > 0$.

- In this case, we show that the desired inverse is known in closed form:

$$z(s) = \left(\frac{z_0}{s_0} + \frac{1 - \frac{z_0}{s_0}}{b_z} \int_{s_0}^{s} \frac{e^{-\int_{s_0}^{y} \frac{x}{a^2(x)} dx}}{y^2} dy \right) s, \qquad s > 0,$$

where $s_0 = \frac{S_0}{\sqrt{M}}$ and b_z is a positive constant: $b_z = \int_{s_0}^{\infty} \frac{e^{-\int_{s_0}^{y} \frac{x}{a^2(x)} dx}}{y^2} dy$.

- This formula shows that distance to default z can be related to the initial stock price S_0 without knowing either the initial asset level A_0 or the promised debt payment L. One must know M & T to get $s_0 = \frac{S_0}{\sqrt{M}}$ & $a^2(\cdot)$.

Assets' Variance Rate Function

- Summarizing Part 2, the local volatility smile $a(s)$ obtained from stock option prices $C_0(K)$ and $\frac{\partial}{\partial T}C_0(K)$ has been used to explicitly express failure probability F and distance to default z as functions of (normalized) stock price $s_0 = \frac{S_0}{\sqrt{M}}$.

- The assets' variance rate function $\eta^2(z)$ is not needed to determine these two credit measures, but it can be numerically determined.

- One can also numerically determine the function relating the assets' \$ volatility to RNDP for each possible level $F \in [0, 1)$ of the RNDP. This can be used to empirically determine whether management's interests are more aligned with the interests of bondholders or the interests of shareholders.

Summarizing Part 2

- By assuming that asset volatility is a function of (just) distance to default, we have show how to analytically calculate RNDP and distance to default as functionals of the stock price and a local volatility smile derived from option prices.
- These formulas for the RNDP and distance to default are both independent of the asset value A and the promised debt payment L. Instead, they depend on the market data described above, along with the time to the debt maturity $M - t$.
- One can also numerically determine the option market's beliefs concerning the relationship between asset volatility and distance to default.

In Conclusion...

- By switching from parametric to non-parametric structural models, one can retain closed-form formulas for failure probability and stock price, while capturing management's flexibility in choosing the asset volatility as a function of failure probability.
- By switching from parametric to non-parametric structural models, one can also analytically invert the capital structure, allowing failure probability and distance-to-default to be analytically consistent with the information content of the stock price and the entire equity options smile.
- By going beyond the information provided by the market prices of just the stock and a single equity option, the firm's failure probability and distance-to-default can be determined without knowledge of the firm's debt level.

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